

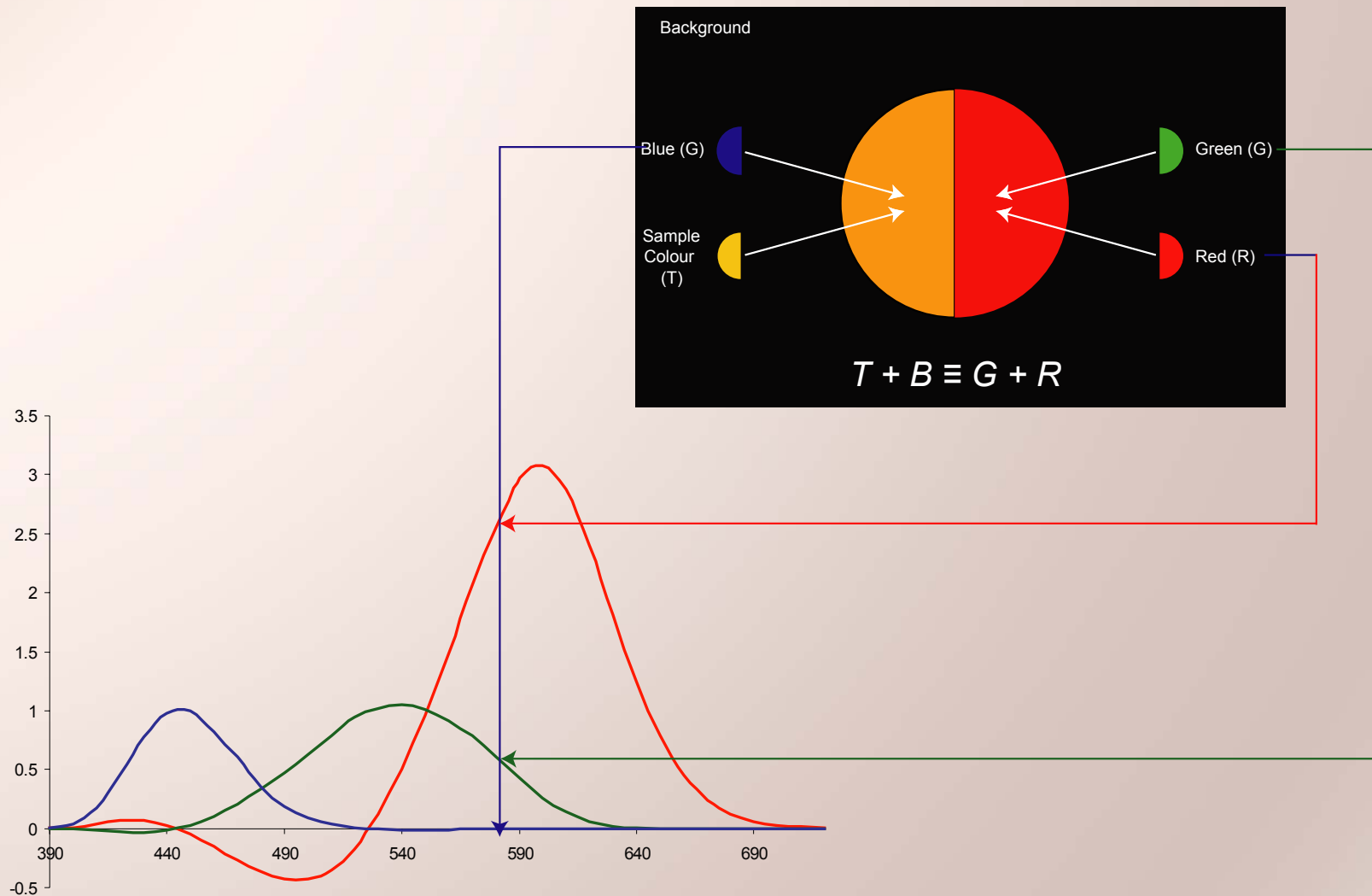


# Test of the transformation of tristimulus space: forward- and inverse-matrix methods

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# Colour Matching Experiment



# Trichromatic generalisation

Symmetry

*if  $A \equiv B$  then  $B \equiv A$*

Transitivity

*if  $A \equiv B$  and  $B \equiv C$  then  $A \equiv C$*

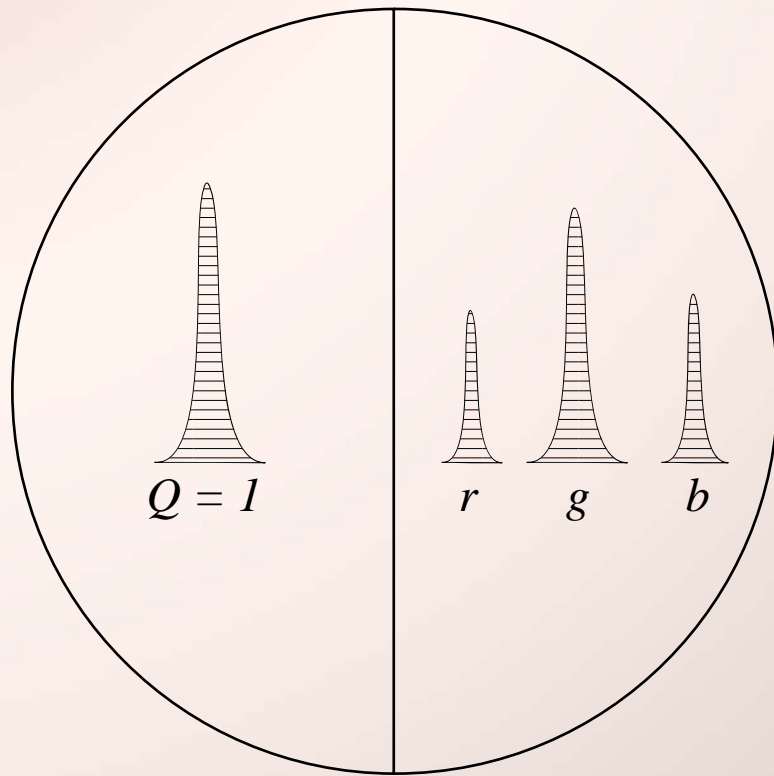
Proportionality

*if  $A \equiv B$  then  $kA \equiv kB$*

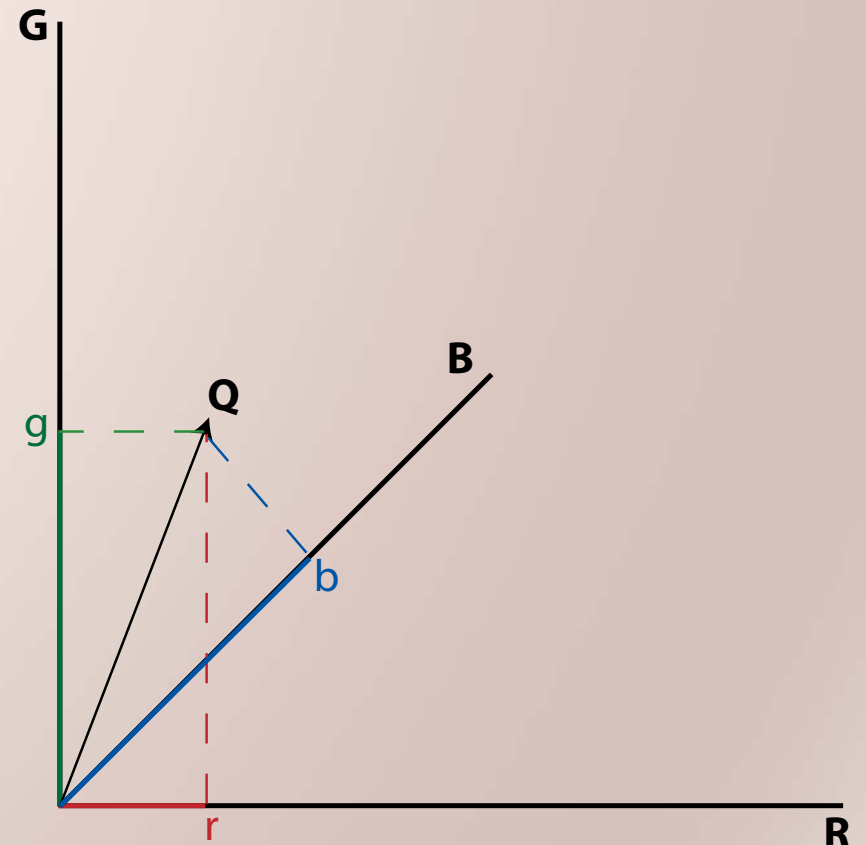
Additivity

*if  $A \equiv B, C \equiv D$  and  $A + C \equiv B + D$   
then  $A + D \equiv B + C$*

# Colour Matching Equation

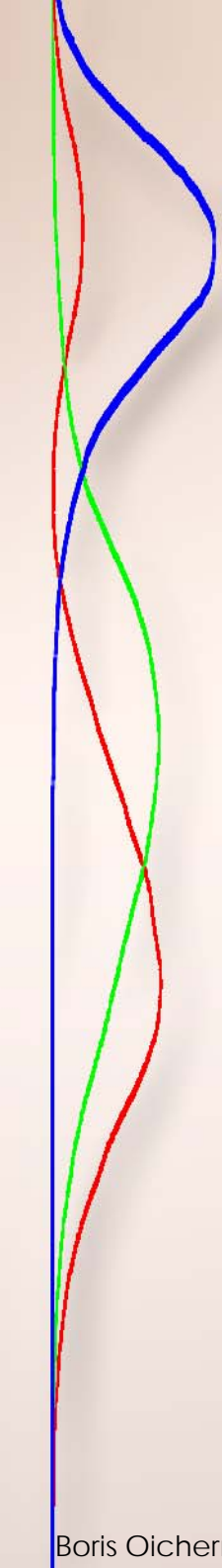


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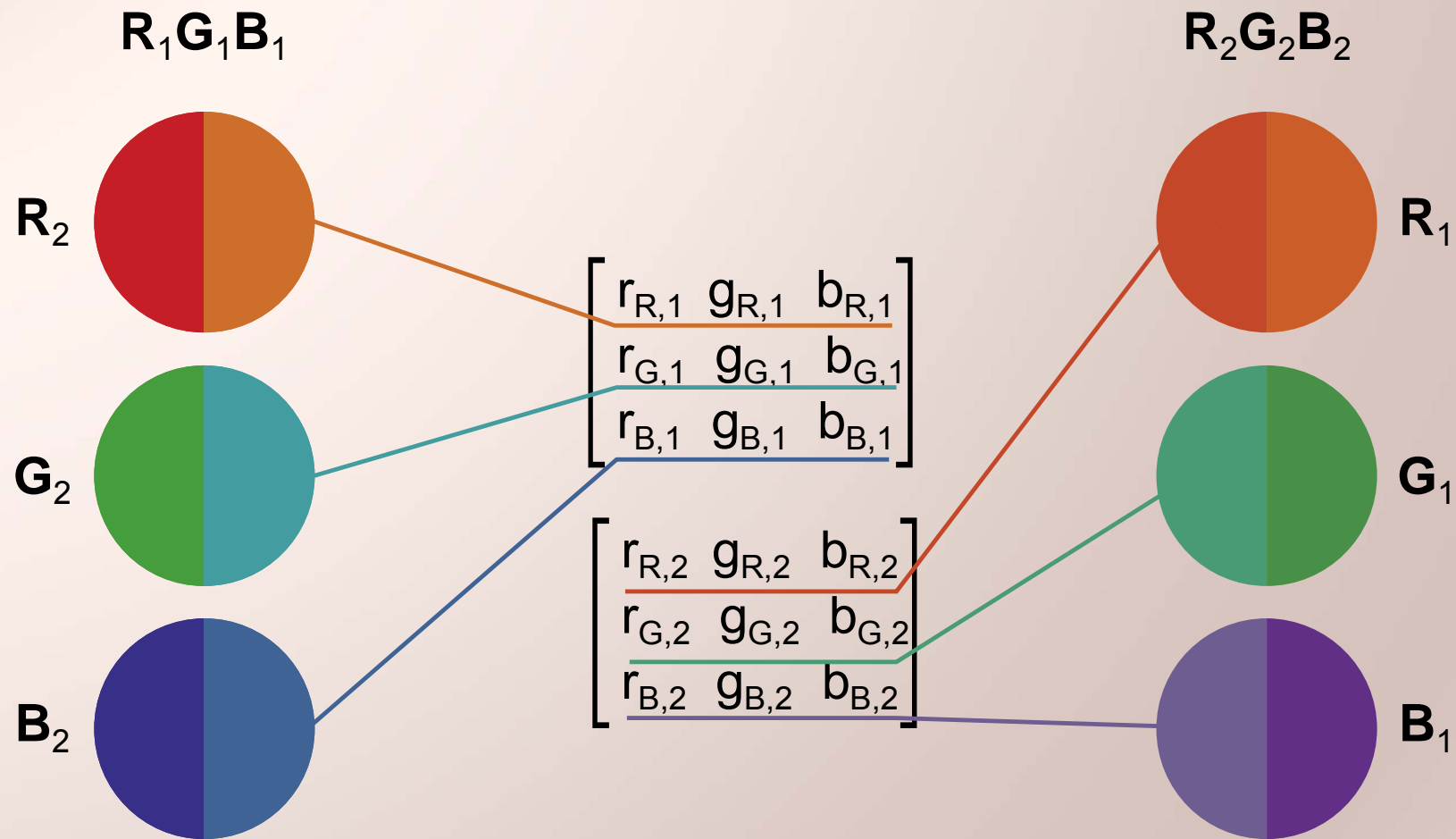


$$Q = rR + gG + bB$$

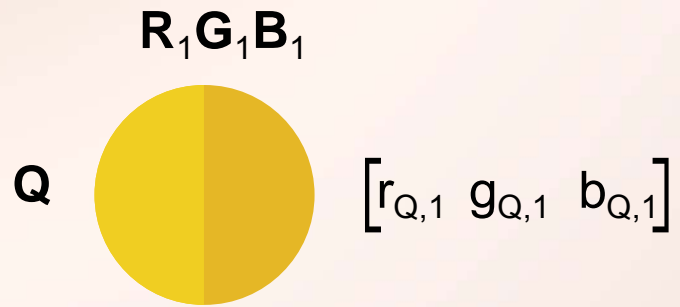
# Transformation of tristimulus space


$$\mathbf{Q} \begin{matrix} \mathbf{R}_1 \mathbf{G}_1 \mathbf{B}_1 \\ \text{Yellow Circle} \end{matrix} \begin{bmatrix} r_{Q,1} & g_{Q,1} & b_{Q,1} \end{bmatrix} \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} = \begin{bmatrix} r_{Q,2} & g_{Q,2} & b_{Q,2} \end{bmatrix} \begin{matrix} \mathbf{R}_2 \mathbf{G}_2 \mathbf{B}_2 \\ \text{Yellow Circle} \end{matrix} \mathbf{Q}$$

# Transformation matrix



# Forward- and Inverse-matrix



Inverse Matrix

$$\begin{bmatrix} r_{R,1} & g_{R,1} & b_{R,1} \\ r_{G,1} & g_{G,1} & b_{G,1} \\ r_{B,1} & g_{B,1} & b_{B,1} \end{bmatrix}^{-1} = \begin{bmatrix} r_{Q,2} & g_{Q,2} & b_{Q,2} \end{bmatrix} \begin{bmatrix} R_2G_2B_2 \\ Q \end{bmatrix}$$



Forward Matrix

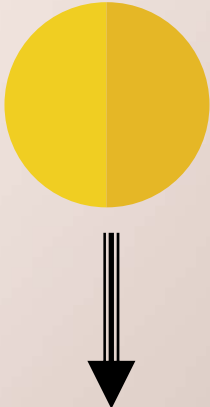
$$\begin{bmatrix} r_{R,2} & g_{R,2} & b_{R,2} \\ r_{G,2} & g_{G,2} & b_{G,2} \\ r_{B,2} & g_{B,2} & b_{B,2} \end{bmatrix} = \begin{bmatrix} r_{Q,2} & g_{Q,2} & b_{Q,2} \end{bmatrix} \begin{bmatrix} R_2G_2B_2 \\ Q \end{bmatrix}$$

# Testing transformation of tristimulus space

Inverse Matrix

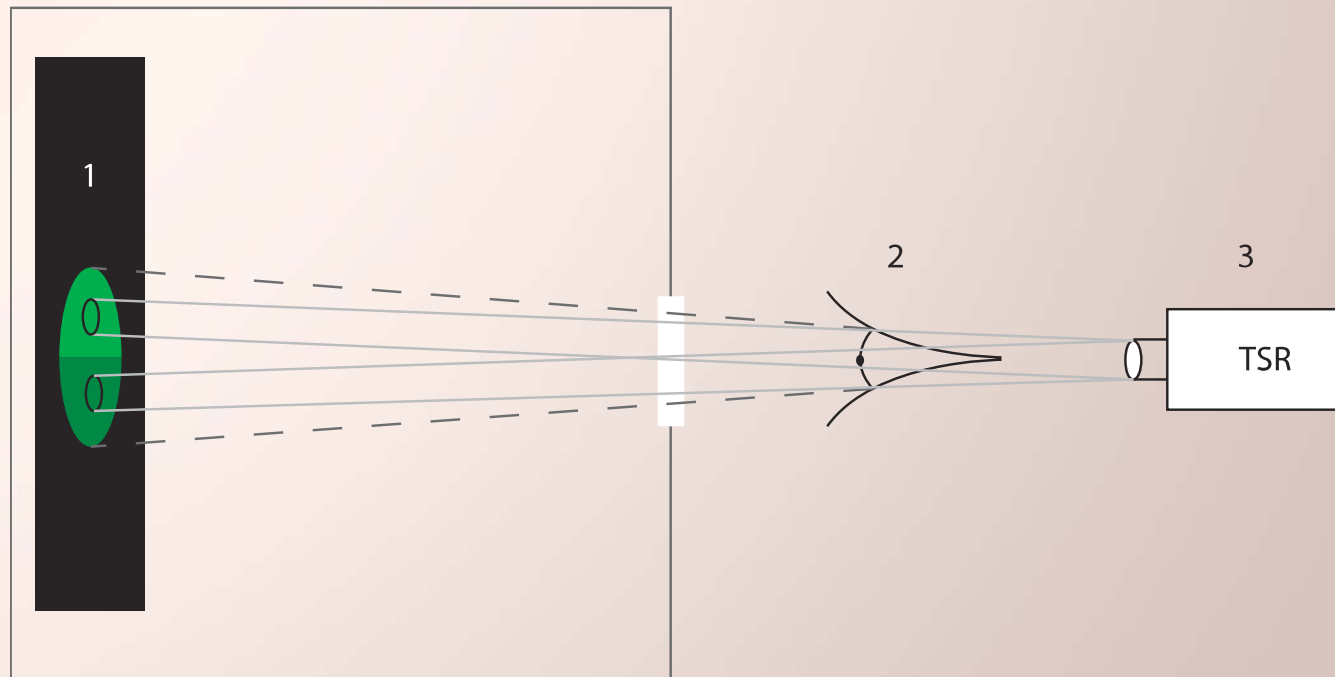
1)  $[r_{Q,1} \ g_{Q,1} \ b_{Q,1}] \begin{bmatrix} r_{R,1} & g_{R,1} & b_{R,1} \\ r_{G,1} & g_{G,1} & b_{G,1} \\ r_{B,1} & g_{B,1} & b_{B,1} \end{bmatrix}^{-1} = [r_{Q,2} \ g_{Q,2} \ b_{Q,2}]_{\text{calculated}}$

2)  $Q \text{ (yellow circle)} = [r_{Q,2} \ g_{Q,2} \ b_{Q,2}]_{\text{measured}}$



$H_0 : [r_{Q,2} \ g_{Q,2} \ b_{Q,2}]_{\text{calculated}} = [r_{Q,2} \ g_{Q,2} \ b_{Q,2}]_{\text{measured}}$

# Experiment: Tarrant visual colorimeter



1: Projection screen

2: Observer

3: Spectroradiometer

# Experiment (cont'd)

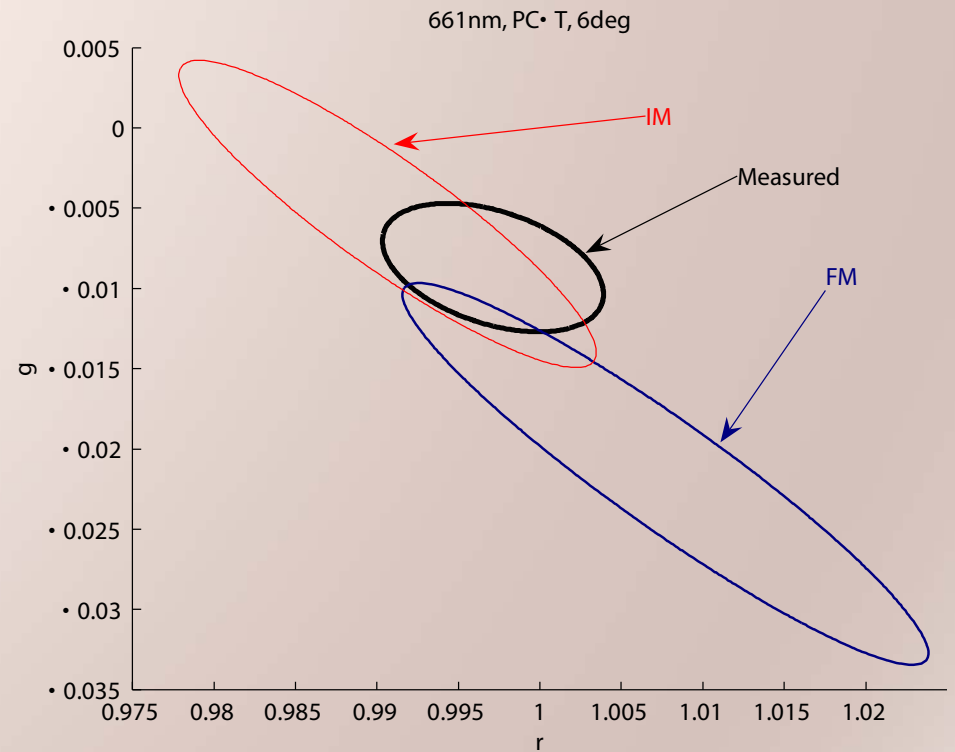
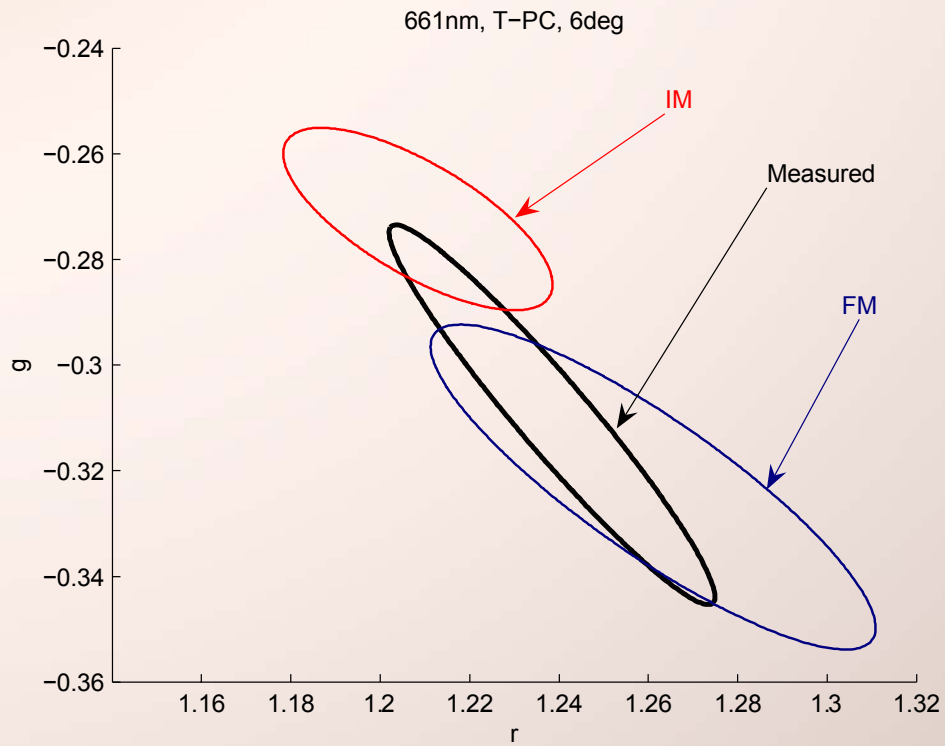
- Observer
  - Male, normal colour vision, 32 years
- Matching field sizes
  - 6° and 2°
- Stimuli
  - 2 primary sets
    - Prime Colours (PC) set: 603 nm, 530 nm, 451 nm
    - Traditional (T) set: 641 nm, 521 nm, 441
  - Test stimuli
    - 461 nm, 541 nm, 661 nm
  - 10 nm bandwidth
- < 125 td
- 10 repetitions of every match

# Results

	PC, 2°, FM	PC, 2°, IM	T, 2°, FM	T, 2°, IM	PC, 6°, FM	PC, 6°, IM	T, 6°, FM	T, 6°, IM
461 nm	√			√				
541 nm						√	√	
661 nm							√	

√ = *"additivity holds"*

# Results (cont'd)





# Conclusion

*"...failure of Grassmann's assumptions III and/or IV in the case of computation of tristimulus values of a single matching pair of doublets, using CMFs from different primary sets, for the same human observer"*

Thornton (1992)



# Additivity failures: historical brief

- **Blottau (1947):** small field
  - Additivity fails for individual observers but holds for average of all
  - Largest additivity discrepancies are in blue primary
- **Trezona (1953, 1954):** small field
  - *“Large deviations from additivity are associated with poor discrimination”*



# Additivity failures: historical brief (cont'd)

- **Stiles (1963)**: large field; calculated tristimulus values vs. visually measured (white)
  - *"...one in three subjects shows a substantial failure of additivity. This takes the form ... of a variation in the ratio of the blue (Z)... to the green (Y) or red (X)"*
- **Crawford (1965) and Wyszecki and Stiles (1982, p. 386)**: small and large fields, Maxwell vs. Maximum Saturation colour matching
  - *"Each observer shows the characteristic difference between the two field conditions..."* (Crawford (1965) )
  - Discrepancies are in green-blue region
- **Lozano & Palmer (1967)**: large field; calculated tristimulus values vs. visually measured (20 colours)
  - *"...three observers out of four were non-additive, especially for the "blue" tristimulus value..."*



# Additivity failures: historical brief (cont'd)

- **Zaidi (1986)**
  - Failures in blue region
  - “... *nonlinearity is due to post-receptoral interactions...*”
- **Thornton (1992)**
  - Tristimulus values are not transformable



# Additivity failures: summary

In the conditions of colour matching experiment conducted at low and moderate luminance levels and with small ( $<2^\circ$ ) and large ( $2^\circ - 10^\circ$ ) bipartite fields, Grassmann's assumption of additivity fails for single observers but holds for the average results of a group of observers.

*"... the extent of the discrepancies and the consequences for practical colorimetry are uncertain"*

*(Lozano & Palmer, 1967)*



# Standard vs. Individual observers

1. CIE system is successfully implemented
2. Additivity holds for the standard but fails for the individual
  - Question: ***What are the practical consequences of the individual non-additivity?***
    - Practical colorimetry
    - Colour Management
    - Cross-media reproduction
    - Image processing



Thank you!