Test of the transformation of primary space: forward- and inverse-matrix methods.

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Introduction

At the basis of the CIE system of colorimetry lies the Trichromatic Generalisation (TG),[1a] which is a mathematical formulation of Grassmann's assumption of additivity, and allows the handling of quantities of colour stimuli in accordance with standard rules of algebra. Assuming the validity of the TG, the result of a colour matching experiment in which a test stimulus is matched in colour by an *additive mixture* of three primary stimuli can be expressed as:

$$\mathbf{Q} = R\mathbf{R} + G\mathbf{G} + B\mathbf{B} \qquad (1)$$

where **Q** is the unit amount of the test stimulus, and its *tristimulus values R, G* and *B* are the scalar multipliers of unit amounts of the primaries **R**, **G** and **B.**

Furthermore, tristimulus values measured with one set of primaries can be transformed to another set by two methods, both equally valid mathematically.[2] The conventional *inverse-matrix* method requires the knowledge of the tristimulus values of the primaries of the destination system in terms of the source system, and can be summarised as

$$\mathbf{T}_d = \mathbf{M}_I^{-1}\mathbf{T}_s \qquad (2)$$

where $\mathbf{T}_d$ is the 3×1 vector containing the tristimulus values of a test stimulus in the destination primary space *d*, $\mathbf{T}_s$ is the vector containing the tristimulus values of the same stimulus in the source primary system *s*, and $\mathbf{M}_I$ is the 3×3 matrix containing the tristimulus values of the primary stimuli of the space *d* measured by means of the primaries *s*. The superscript "-1" represents the matrix inverse operation.

An alternative method develops if the tristimulus values of the primaries of the source system in the destination system are known:

$$\mathbf{T}_d = \mathbf{M}_F\mathbf{T}_s \qquad (3)$$

where $\mathbf{M}_F$ is the 3×3 matrix containing the tristimulus values of the primary stimuli of the space *s* measured by means of the primaries *d*. This method is denoted as *forward-matrix* in the following text.

Two maximum-saturation[1b] colour matching experiments have been set up to test the TG for the case of single observer using small (2°) and large (6°) fields. Both transformation procedures (Equations (2) and (3)) were used. Apart from the field size, the experimental conditions in the new 2° experiment were identical to the previously reported 6° study.[3] Results concerning the validity of the assumptions of additivity are reported, along with a discussion of the consequences of the choice of the mathematical method of primary space transformation.

Results and Discussion

Two sets of narrow-band primary lights were employed: one similar to the Prime Color[4] (PC) set, and one similar to the final set used by Stiles and Burch[5] in the experiment that led to the 1964 Standard Observer (T, for traditional). One colour-normal observer performed ten repeated matches of each of six stimuli, thus allowing intra-observer variability to be taken into account in analysing the results. The six stimuli were the three primary stimuli of the other set and three "test" narrow-band stimuli: 461, 541 and 661 nm. The matches of the primary stimuli of the other set were used to construct the transformation matrices $\mathbf{M}_I$ and $\mathbf{M}_F$ (Eq. (2) and (3)), which were then used to transform the tristimulus values of the test stimuli between the primary spaces. The Null Hypothesis that the additivity assumption holds in the conditions of our experiment was tested by the statistical T test (at 95% confidence level) of the equality of the tristimulus values measured directly and the values predicted by the models (2) and (3).

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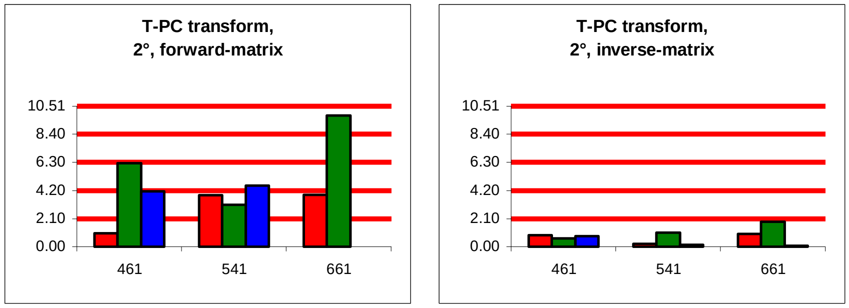

Figure 1: Results of the t-test statistics for each tristimulus value of each test stimulus for the 2° field and two transformation methods. The abscissa indicates the test stimulus in nm; the ordinate indicates the 95% critical values from the t-distribution table. A bar crossing the first gridline signifies a failure of additivity.

The results for the small field and the T-PC transformation are illustrated graphically in Figure 1, which shows a striking difference in the results of the analysis of the same 2° data by the two methods of transformation. When the traditional inverse-matrix method is employed, the results indicate no statistically-significant failure of additivity. With the forward-matrix method, however, the transformation fails for two or more tristimulus values of all three test stimuli. The results for the PC-T transformation for the small field and for both transformation methods for the large field differ slightly in details, but show similar trends.

Numerical analysis of the outcomes of the two procedures reveals that the major cause for the differences in the results they produce is the inflation of experimental uncertainties in the process of the inversion of the matrix $\mathbf{M}_I$ (Eq. (2)). In other words, the results illustrated by the right-hand chart of Figure 1 show no additivity failure merely because of the larger uncertainties entering the Null Hypothesis test, and not because the failure itself does not take place.

Additivity failures have been reported by numerous researchers;[4,6-8] however, there is no general agreement for their possible causes. Zaidi[7] performed perhaps the most comprehensive test of additivity so far; he found intra-observer failures of additivity mainly in matches made in the short-wave region of the spectrum, and concludes that they could be caused by post-receptor processing. The results of the present experiment also indicate statistically significant failures of additivity. However, they also indicate that the choice of the calculation procedure can have a dramatic effect on the conclusions. The consequences of this choice need to be further investigated.

References

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