A New Method For Calibrating Colorimeters

Maria Luisa Rastello
Istituto Elettrotecnico Nazionale Galileo Ferraris
Torino, ITALY

Introduction

This paper discusses a new method for calibrating colorimetric devices with a devoted Least- Squares approach, allowing the uncertainty to be taken into proper account.

Let us consider two different colorimetric devices and try to calibrate one with respect to the other. A typical example is to compute the CIE 1931 XYZ tristimulus co-ordinates from the response of a colour camera. Let subscript R denote the reference device and D the device under test, and M be the number of sensors in both devices.

Now let us suppose to have $N > M$ test colors as input to both devices. The output signals from the devices will be denoted by $\Gamma_R \equiv \{\Gamma_{Ri,n}\}$ and $\Gamma_D \equiv \{\Gamma_{Di,n}\}$, respectively, with $i = 1,2,\dots M$ and $n = 1,2,\dots N$. Taking into account that both Γ_D and Γ_R are affected by errors $\tilde{\Gamma}_D$ and $\tilde{\Gamma}_D$ with zero averages and different variances, more generally we have

$$\Gamma_{Ri,n} \to \Gamma_{Ri,n} + \tilde{\Gamma}_{Ri,n} \qquad i = 1,2,\dots M \quad n = 1,2,\dots N$$

$$\Gamma_{Di,n} \to \Gamma_{Di,n} + \tilde{\Gamma}_{Di,n} \qquad i = 1,2,\dots M \quad n = 1,2,\dots N$$

To calibrate the device *D*, we determine a linear transformation *C* such that

$$\Gamma_D \cdot C = \Gamma_R \qquad (1)$$

The conventional Weighted Least Squares (WLS) techniques cannot be applied to solve the over-determined system of linear equations (1), as they assume that independently distributed (i.d.) errors affect only the known terms Γ_R. So, we apply the Least Squares formulation with Element-wise Weighting, hereafter called Element-wise Weighted Least Squares (EWLS), to solve the problem in a appropriate way.

The Element-wise Weighted Least Squares method

EWLS considers linear models described by a linear algebraic system of equations $AX = B$. Here $D := [A, B]$ contains the *measured data* and X is the *parameter* matrix, to be estimated. With less parameters than equations and with noisy data the model equations cannot be exactly satisfied, the residual matrix $R = AX - B$ is considered, and an approximate solution for X is sought.

The classical least squares (LS) approach minimises the *Frobenius norm* of the residual matrix, by applying the correction ΔB with the smallest Frobenius norm to the right-hand side B in order to make the corrected system exactly solvable. The LS method is the best linear unbiased estimator when *A* is noise free and *B* is corrupted by independent and identically distributed (i.i.d.) errors.

The total least squares (TLS) technique is a parameter estimation technique for the linear model when all elements of D are perturbed by i.i.d. errors. In this case, a correction $\Delta D = [\Delta A \, \Delta B]$ is applied on D, so that the corrected system of equations

$(A_0 + \Delta A)X = B_0 + \Delta B$ becomes exactly solvable. Again the smallest correction, according to the Frobenius norm, is sought. The TLS approach requires that all variances are coincident, so in some sense TLS requirements are more strict than WLS ones, where the variances of each known term can be chosen independently, and consequently fit more difficultly to statistics of actual data. The generalisation for the case when the errors are independent but not identically distributed with element-wise different error variances is called *element-wise weighted total least squares* [1].

Solving the EWLS problem consists in finding the optimal values of the problem variables X and ΔD minimising the cost function

$$\min_{X,\Delta D} \sum_{i=1}^{m} \Delta d_i^{\mathrm{T}} \left[V_i^d\right]^{-1} \Delta d_i \qquad \text{subject to} \qquad (D + \Delta D)\begin{bmatrix} X \\ -I \end{bmatrix} = 0$$

where $\Delta D \equiv [\Delta d_1 \quad \Delta d_2 \quad \cdots \quad \Delta d_m]^{\mathrm{T}}$ are the correction on measured data to compensate for the measurement error $\widetilde{D}$, random matrices defined as $\widetilde{D} = [\widetilde{A} \quad \widetilde{B}]$, with zero mean and independent rows $\widetilde{d}_i$ with known $m \times m$ covariance matrices V_i^d.

An example: the calibration of tristimulus colorimeters

Traditionally, for the calibration of a tristimulus head, the CIE illuminant A is recommended, as it is characterized by chromaticity coordinates $\{x_{\mathrm{A}}, y_{\mathrm{A}}, z_{\mathrm{A}}\} = \{0.44758, 0.40745, 0.14497\}$ known *a priori.* Provided a tristimulus head with $M = 3$ channels, x, y, z, is illuminated with a CIE illuminant A source, photocurrents V_x, V_y, V_z are measured. If multiplied with the calibration factors c_x, c_y, c_z the tristimulus values X, Y, Z are found. The values assigned to the calibration factors are found from the solution of the exact equation system defined by

$$\Gamma_D \cdot C = \Gamma_R$$

where $$\Gamma_D = \begin{bmatrix} V_x & 0 & 0 \\ 0 & V_y & 0 \\ 0 & 0 & V_z \end{bmatrix}, \quad C = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix}, \quad \text{and} \quad \Gamma_R = \begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix}.$$

Calibration factors are simply given by $C = \Gamma_D^{-1} \cdot \Gamma_R$. If the device under test is only to be used for measurements on tungsten based sources, then the calibration could give good results. If this is not the case, you can get odd results. To overcome these drawbacks, the here-proposed method makes use of $i = 1, \ldots, N > M$ test colors and solves the over-determined equation system (1) in a EWLS environment, where

$$\Gamma_D = \begin{bmatrix} \cdots & \cdots & \cdots \\ V_{xi} & V_{yi} & V_{zi} \\ \cdots & \cdots & \cdots \end{bmatrix}, \qquad C = \begin{bmatrix} c_{xx} & c_{xy} & c_{xz} \\ c_{yx} & c_{yy} & c_{yz} \\ c_{zx} & c_{zy} & c_{zz} \end{bmatrix}, \qquad \Gamma_R = \begin{bmatrix} \cdots & \cdots & \cdots \\ X_i & Y_i & Z_i \\ \cdots & \cdots & \cdots \end{bmatrix},$$

and covariance matrices are taken into account appropriately. Experimental results will show the capabilities of the proposed method for accurate calibrations.

[1] Ivan Markovsky, Maria Luisa Rastello, Amedeo Premoli, Alexander Kukush and Sabine Van Huffel, *The element-wise weighted total least-squares problem*, Computational Statistics & Data Analysis, Volume 50, Issue 1, 10 January 2006, Pages 181-209