# Johann Heinrich Lambert's *Farbenpyramide*

**Translation of**

## Beschreibung einer mit dem Calauischen Wachse ausgemalten Farbenpyramide (Description of a color pyramid painted with Calau's wax)

**With a brief introduction and biographical information on Lambert and Calau**

# Introduction

Johann Heinrich Lambert's *Farbenpyramide* is an iconic document in the history of the development of color order systems. Before Lambert there was the theoretical color mixture concept of Franciscus Aguilonius of 1613, the named tint/shade scales of Johannes Zahn of 1685, the 12-hue circle of the anonymous author of a chapter on pastel painting published in 1708, and the theoretical double tetrahedron by Tobias Mayer of 1758.

Lambert had a restless mind and was engaged with many subjects as his biography and list of publications show. In 1760 he published a seminal book on the measurement of light, *Photometria*, in which colors also play a role. In 1770 he presented a lecture on light measurement as related to art. In 1758 he had visited Göttingen and hoped to get an academic position at its university. It is likely that during his visit he also met Mayer who then was a professor at the university. It is not known if they discussed color as both of them also had great interest in astronomy. Lambert does not mention a meeting in *Farbenpyramide*. In its Chapter 4 he copies the 1758 newspaper report about Mayer's lecture from *Göttingische Anzeigen* that he had read in the same year. In 1762 Mayer succumbed prematurely to an illness. Mayer's triangle is mentioned in Lamberts 1770 lecture at some length. At the same time he indicates that he does not believe that further work on the subject will be done in Göttingen.

Lambert then decides to theoretically and practically investigate Mayer's proposal in some detail. The result is *Farbenpyramide*. A number of important issues arise, such as what colorants should represent the three basic colors, how the relative strength of the selected colorants can be determined, how to deal with lightness in the mixtures, and others. Lambert arranges for the court painter Benjamin Calau, recently relocated to Berlin, to assist him with the project. Lambert the mathematician also plays a significant role in it. Mayer assumed that the central color in his basis triangle was a middle gray. When making trials Lambert and Calau soon found that equal mixtures of all three of the basic colors they selected were very dark. Even though they had determined the relative strength of all three by determining in pairs of two the relative amounts required to obtain a color perceptually half way between the two basic colors, the most nearly black colors were not located in the center of the basis triangle but closer to the blue corner. As a result, there is no central gray scale even though Lambert had already earlier discussed a 30 grade gray scale. In addition, the use of Calau's wax, while intensifying chroma, also darkened colors by making them glossier. The collision between theory and reality is evident, and it is of some interest to learn how Lambert dealt with them. How he did so can make reading this translation useful to colorimetrists as well as to artists.

A few explanations/comments have been added in footnotes or in [ ].

# Johann Heinrich Lambert (1728 – 1777)


Lambert was born into a family originating in Lorraine. Having turned Calvinist the family fled in 1635 south to the independent city of Mulhouse, just north of the Swiss and west of the German border. Mulhouse had joined the Swiss Confederation in 1555 as an enclave, rejoining France during its revolution in the early $19^{th}$ century. Lambert's parents were Lukas Lambert, a tailor, and Elisabeth Schmerber. Johann Heinrich had two sisters and four brothers. He went to school in Mulhouse until age 12, learning French and Latin in addition to the usual subjects. It was then that he had to leave formal schooling to begin assisting his father in his business, continuing intensive studies in his spare time. To earn more money and as a result of his aptitude in calligraphy he moved on to a job as a clerk. By age 17 he assumed the job of secretary of a newspaper publisher in nearby Basel, Switzerland. He also began to work as a private tutor. At age 20 he became the tutor of three boys in the family of Count Peter von Salis in Chur, Switzerland. There he had access to the count's large library and was able to widely travel in Europe with his charges. In 1755 he began to publish scientific articles on a number of subjects. In 1756 he travelled with his pupils to Göttingen and was elected to the *Königliche Gesellschaft der Wissenschaften* (Royal Society for the Sciences). In 1758

Lambert left von Salis's employ and tried to find an academic position. In 1759 he published his work on light measurement, *Photometria*, and *Cosmologische Briefe* (Cosmological letters). In the former, he introduced his mathematical formula for the law of the absorption of light (Lambert's law), described non-mathematically a few years earlier by Pierre Bouger. In 1764 he followed an invitation by the mathematician Leonhard Euler to come to Berlin where, after some initial difficulties, Frederic II appointed him to a position in the *Königlich-Preussische Akademie der Wissenschaften* (Royal Prussian Academy of Sciences). Among many other achievements, Lambert was the first to mathematically prove the irrationality of the number π. In 1770 he published an article *Mémoire sur la partie photométrique de l'art du peintre* (Dissertation on the photometric component of the art of the painter) in which he discussed the effect of light on the appearance of colored materials.

Lambert, as mentioned, had read the report on Tobias Mayer's public lecture on a color order system in *Göttingische Anzeigen für gelehrte Sachen* (Göttingen reports on learned matters). The follow-up expected by Lambert did not take place due to the untimely death of Mayer in 1762. In the wake of his occupation with the subject of color and art leading to his 1770 article Lambert decided to produce an example of Mayer's system. However, practical matters, as discussed in *Farbenpyramide,* resulted in a single triangular pyramid, the subject of the present translation.

In 1775 Lambert became ill with an infection, refused treatment, and passed away in 1777. One of the craters on Mars is named after him.

Lambert was an unusual character, an individualist who did not always live up to the rules of the society he lived in, in regard to behavior as well as clothing. Some people in Berlin called him "a man from the moon." When he became absorbed by a mathematical problem he could go mentally absent from his surroundings for hours. An anecdote told by the mathematician Daniel Huber (1786-1829) is that while Lambert was working on the subject of light reflection he saw the need for some experimental facts: "He required a large mirror and for this reason entered Berlin's most famous Caffeehaus. There, several officers and a few citizens played cards. As it was his custom, he greeted them without looking at them, turning his head to the right side and immediately placed himself in front of the large mirror. He pulled out his rapier, advanced and retreated, made all kinds of movements as if he was dueling, and thought for a while about what he had seen. This he did for half an hour without noticing that all those present, not realizing what was going on and taking him for a fool, surrounded him and got ready, should it become necessary, to disarm him. After he completed his tests and observations he replaced the rapier in its scabbard, looked calmly at those around him, greeted them just as when he arrived, and returned home to write his related treatise." [1]

Lambert was a leading mathematician, astronomer, physicist, and philosopher of the 18$^{th}$ century. The complete works of Lambert are available online at www.kuttaka.org/~JHL/L1772b.html.

## Benjamin Calau (1724 – 1785)

Born in Germany, Calau was educated as a painter by his father Christoph Calau and followed him in 1743 to St. Petersburg, Russia, where he was named an imperial court painter. In 1746 he returned to Germany and spent time in Leipzig and Dresden. In 1771 he was named Prussian court painter. While in Leipzig he began to engage in the ancient Roman practice of encaustic painting, painting with wax. In 1769 he published in Leipzig a pamphlet "Ausführlicher Bericht wie das Punische oder Eleodorische (?) Wachs aufzulösen" (Detailed report on how to dissolve Punic or Eleodoric (?) wax). In the last years of his life Calau became a manufacturer of wallpaper.
Examples of Calau's work, primarily portraits, executed in various techniques are found in a number of German museums.

[1] Huber D., *J. H. Lambert nach seinem Leben und Wirken*, Basel: Schweighauser, 1829, p. 37.

## Description of a color pyramid painted with Calau's wax, where the mixture of any color from white and three basic colors is arranged, explained, and its calculation and multiple uses demonstrated by J. H. Lambert.

**With an illuminated copper engraving. Berlin: Haude & Spener, 1772**

### Introduction

Most of what could be mentioned here in the introduction can be found in the text in much more coherent form. This includes the reason for and the history of this undertaking as well as, to a degree, the subject itself. It is concerned with that part of coloration or the art of color that has as its subject the richness of colors in their universal harmony, as far as colors are colors and each loses itself into the next one bordering it. The purpose of the book becomes clear at first glance on its illuminated copper engraving. It presents a pyramid or little box drawn in perspective, separated into compartments where each compartment contains the colors that belong there represented by the corresponding number of tiles. It will be up to the reader to determine if he wants to produce such a pyramid or color box. Interested people do not need to concern themselves about how to purchase all these colors. The only colors necessary are the three corner colors in the lowest compartment. They are carmine, Berlin blue, and gamboge. All other colors in each compartment, including those that are carbon- and pitch black are just mixtures of these three colors, while paper served in place of white. How the mixtures must be produced does not belong into the introduction but is part of the main text, where it will be described in detail and where the proportions in each mixture are calculated. What should be mentioned immediately is that the colors located on the bottom frame of the pyramid or color box are not members of the colors of the pyramid, but are other painter's colors, placed there for comparison. The last chapter of the text provides detailed information for what purposes the painted pyramid, in addition to being a very general and naturally ordered collection of colors, can be used. But it is recommended to first read the earlier chapters. They already contain much information about the purpose and use of the system, to the extent allowed by the theoretical considerations. It will also become obvious why the pyramid has this specific form and not another one, and the reader can more easily determine what use he might make of the system later on. Calculation in the 12$^{th}$ chapter more extensive than normally found in instructions for the art of painting was unavoidable; this cannot be different in a book concerned with accurate and mathematically correct knowledge of all steps of color mixture. So far such knowledge was particularly lacking in the theory of coloration. How to go about producing a particular mixture of colors not in hour-long trial and error but immediately and as often as one wants, according to certain and proved rules, has been a question never fully addressed. The history of this problem with many related comments runs pretty much through the complete text, even though most of it is present in chapters 3 to 6. There is also considerable additional material discussed that, when further investigated, will in the future enrich and continue the subject matter. Until then what is shown in this work can be used for many good purposes as it already has for some. It will become obvious when reading chapter 5 that Calau's wax, mentioned in the title, is a very special product that has nothing to do with the wax painting method described by Count Cailus, and is not just a wax related to oil according to Taubenheim, nor a wax allied with water, with alcohol, with various liquids, with email, with any of the varnishes, etc., will become evident from the information given in chapter 5. There it will also become evident why it has been used in the colors of the color pyramid presented here.

## Table of Content

## Chapter 1 Common differences present in every color

§ 1

Nature's richness of color offers us a multitude that in several respects touches on infinity. Many a poet has taken the opportunity to loose himself in this infinity and to represent charming objects in a delightful manner also from the point of view of their splendor of colors. This is not our intention here. We want to leave for the poet an infinity of colors of various kinds and not limit his powers of imagination in this regard. He may find them as infinite as the sand of the sea, the stars in the sky, dust in the air, or dewdrops on the flowered meadow. It is not his duty to calculate the extent of what he calls infinite and unlimited. Often a quite limited number already suffices. He is also quite permitted to expand the number as he sees fit and place millions or, according to the modern style, myriads where, when accurately numbered, barely a few hundred are found. The thus boasted about uncountable army of visible stars does, even in the clearest night, not exceed a thousand by much and if one wants to consider not half but the whole sky and count altogether 3,000 it is necessary to use a telescope. The uncountable sand did not limit Archimedes to claim a number that is larger than the number of all sand corns of the world by a factor as large as desired. The unlimited multitude of colors does not restrain us from viewing it closer to determine how one can possibly count them. The nature of colors already contains the fact that their infinity is due to infinitely small steps and thereby much of the apparent difficulty disappears. The most extreme limits are not infinitely far from each other and thereby we obtain units with which any particular parts can be compared.

§ 2

We begin by separating colors according to the most common differences and thereby get into a position to place them accordingly in different classes. I say: the most general differences. They are those that occur in case of every color. In colors there are only a few: 1. **Glossiness**, 2. **Strength**, 3. **Vividness**, and the opposites 4. **Dullness**, 5. **Weakness**, 6. **Deathliness**. I take the meaning of dullness as the opposite of glossiness and it is apparent that it represents complete lack of gloss, thereby being its lowest value or zero. Glossiness itself relates only to the, so to speak, polished surface of the color. Accordingly, a matte-polished surface of a multicolored marble shows color without gloss. The colors become glossy to the degree the marble surface is flattened or polished. The oil of oil paints, the binder in water colors, cream of tartar in verdigris, as well as the varnish in any color offer the same service in regard to the appearance of gloss, to a degree also glass placed in front of a painting. Colors made glossy in this manner must, of necessity, be viewed in a special light. They are, like a kind of mirror, those that reflect light. If this reflected light arrives through a window, from the sun, or at night from lamps, oil or candles, or from a flame it must not directly fall into the eye, because then the true color cannot be perceived. Therefore, both the painting as well as the viewer must be placed so that the colors can be seen as they are and without the reflected light. However, the colors may be augmented themselves if the reflected light arrives from an object of the same color. But this happens only very rarely.

§ 3

Above, I related **glossiness** in colors only to their surface, in case it is flat and reflects the incoming light. The purpose is to distinguish it from another kind that I will call **vividness.** There are colors that reflect light not just from the particles on the surface but also from particles in the interior, even when the surface is not flat. The light thus reflected has mostly the color of the object. It therefore is uniform and in this manner approaches the vividness experienced in prismatic colors. It is easy to understand that this can only take place where color has a certain degree of transparency, as in juices. This also demonstrates what is necessary to have true **lushness** in colors. It depends on the degree of transparency. It makes it possible for a color, even without glossy surface, to have lushness. Transparency also gives colors increased vividness, because the colored rays are not just reflected from the surface but also from the interior, thereby making a stronger impression on the eye. This effect is in part due to a uniform mixture of rays of the same kind. The depth from which colors are reflected from the interior particles is very different for different solids and colors. It also determines the degree of transparency. This depth is very considerable in case of colored glass. Even in case of the densest substance, gold, it has at least the thickness of a gold leaf because, according to **Newton**'s observation, it has a degree of transparency. In case of earth colors it is usually very small, but larger in case of colors of plant juices. It is for this reason that juice extract colors are more lively, while earth colors are usually dead and in oil painting have a more deadly appearance the more the oil dries and loses its transparency. Gloss and vividness are thereby slowly lost if the color itself does not have some of it. Varnish contributes to maintain in the colors not only gloss but also lushness and vividness, to the degree that the painting is covered with it varnish and oil maintain transparency. In case of water colors the binder provides a comparable service. The binder itself is transparent, and the more transparent the better. It thereby increases the transparency of juice-based paints and even provides a degree of transparency to the dead earth colors. However, it often induces in colors an unnecessary and inappropriate gloss, also induced by oil and varnish. In this respect the Punic wax rediscovered by the Berlin court painter **Calau** is a much superior medium, especially for earth colors because it induces in the basically dead colors more liveliness and lushness and maintains both very well. Vividness of colors is often co-mingled with lightness. But this is not whereof I speak here. Regardless of the lightness lighter colors, even white, can have little vividness as well as little gloss. A light color reflects in addition to its specific colored rays always more or less white light, co-mingled with the colored, and is in this regard not of the same kind, but mixed with white. A special terminology is used to describe an increased degree of vividness. For example, red is called burning or dazzling, burning or dazzling blue, etc., even if the red or blue colors are noticeably dark. Such higher degrees are shown in nature not only in flowers, butterflies, and several often tropical birds, snakes, etc., whereby not only transparency and lushness, but in part to a degree also something truly prismatic occurs, in the latter case especially where colors change as a result of light, position, and location of the eye.

§ 4

The strength of colors is different from their gloss as well as from their vividness and derives from the degree of packing of color particles on the surface. This is the reason that it is highest in case of metals so that they can be imitated neither with earth nor with juice extract colors other than approximately because they are less dense. When metals are dully polished, one can observe the true strength of their color. With further polishing they gain glossiness and vividness is added when they are covered with a varnish of the same color, or when placed in fire where e.g. steel takes on a bluish color while Dutch ducat coins take on a more intense and reddish orange color. Gold bugs have nearly the vividness and gloss of gold but not the total strength or denseness of the color. Painter's colors of greater strength offer considerable yield when painted and cover the background well. Among them are carmine, cinnabar, smalt, Berlin blue, auripigment, etc. On the other hand, juice extract colors must often be applied generously if they are to have a solid degree of strength. In this way they usually also obtain a degree of vividness. If earth colors are over-painted with juice extract colors they obtain a kind of glaze and thereby more vividness and show to a greater or lesser extent transparency. Such effects have excellent application when painting grapes, cherries, berries, etc, as well as in case of polished metals, to the degree that the images of such objects are visible.

§ 5

It is evident from what has been said so far that the three explained attributes of colors can be present singly or all of them together. Each of them has multiple grades and each grade of one can be combined with each grade of the other two. However, the grades do not reach into infinity but only to one that we consider to be unity. The lesser grades can be represented as fractions. A glossy color cannot reflect more light than what impacts on it. But a color of this kind does not exist in nature. The surface of purest mercury reflects only 2/3 of the impacting radiation (see *Photometry*, § 687). Polished pewter or copper reflect much less, in part because they are softer and less dense. The closer a glossy color approaches black the less light it reflects. However, much depends also on a smaller or larger angle of the incoming light. If it is only two or three degrees, most of the light is reflected, regardless how blackish the polished material is. Vividness and strength of colors have comparable scales. The former cannot exceed the state where all rays of the incoming light reflected by the color of the object are returned from the surface and the internal particles, the remaining colored rays losing themselves in the object. This case also does not happen in nature. But it indicates the unit of which the reduced grade of vividness is a fraction. Finally, also the strength of colors cannot be unlimited, because there is no infinitely dense solid. Therefore metals provide the highest real degree of denseness. Metallic colors, such as smalt, azurite, royal yellow, etc., are closest to them. Juice extract colors are generally less strong. By the way, I have already indicated in *Photometry* the means by which these scales can be determined more exactly. As reported there (§ 747 ff.) I have found by experiment that a white paper, or one painted with Cremnitz white has a value of 2/5, one painted with red lead 3/10, painted with cinnabar 3/10, with azurite 1/7, with royal yellow 39/100, one strongly painted with the juice of berries of *rhamnus catharticus* ¼, one painted with verdigris and a green extract 1/11 of reflection of the incoming equally-colored rays. The green paper used in this case was neither dull nor dead, even though it was painted very densely or strongly with color.

§ 6

Earlier I said that gloss, strength, and vividness are attributes that can be present in each color, and at the same time reminded the reader that I distinctly differentiate vividness from lightness, as far as the latter refers to whiteness, because they are different properties. I do not regard a color that tends toward white or black, or in the direction of any other color, as less or more of a color because, regardless of how much it is mixed it can always have a degree o gloss, vividness, or strength. These properties allow the color itself or its category to be what it is. This is important to keep in mind when one wants to count the colors, with introducing confusion into its categories.

## Chapter 2 Influence of external light on colors

§ 7

I will touch on only one more subject that also concerns every color but is not in the colors but outside of them. It concerns the light with which colors are illuminated. It should be white so that every color can reflect the colored rays related to it. Sun light tends toward reddishness the closer the sun gets to the horizon. If it is higher in the sky the light is stronger and whiter. On white paper moonlight shows a milk color, to which the light bluish air visible next to the moon contributes. The light of lamps tends toward yellowish red. The light from white wax is much brighter. The most accurate comparison is made in a manner described in *Photometry* § 1075 where I experimentally found that an oil lamp has slightly more than twice the brightness of moon light when both are compared by naked eye, the moon is high and the air maximally transparent and one takes an average between the brighter and darker regions both of the flame as well as the moon.

§ 8

The earlier mentioned difference in the color of the illuminating light contributes considerably to change the shine of colors illuminated by it. Everybody knows that at night in lamp light blue and green colors are not easily distinguished. During the day when there is a bright and strongly blue colored sky a white wall sitting in shade can easily appear blue-gray or even fully blue. This is best observed in a dark room with the image of two walls, one of which is lit by sun light, the other by the blue sky, projected onto white paper. A blue sky can make a blue color appear more strongly blue, a yellow-green one completely green, a brownish black one completely black, a dark red-yellow one brown, a red one more or less violet. Evening and morning redness has similar influence on red colors. It changes blue towards purple, yellow toward orange, the latter toward lead red or cinnabar, etc. There are comparable mixtures of colors in case of clothing, particularly where there are folds or where one piece of clothing is close to another one and reflects its color on the other one. These induce the painter to be concerned less with the colors of objects and more with those resulting from illuminating colored light falling on the complete object; without such detailed knowledge they cannot represent velvet as velvet, taffetas as taffetas, damask as damask, and polished metal vessels as such. Results of specific related experiment are also reported in *Photometry*. They indicate that **red** and **blue** went through all stages of violet and purple depending if redness and blueness had stronger light intensity and higher density of rays. **Red** and **yellow**, **yellow** and **green**, as well as **green** and **blue** produced the intermediate colors quite well. **Green** and **red**, however, produced a dead, brown-gray color of excrement, blue and yellow different dark gray, mouse gray, rust colored, mud colored and gray-green colors, the latter probably due to the fact that the yellow was slightly reddish and also the blue not a totally pure one (Photometry § 1190-1195). With a better blue and yellow I obtained in a comparable test a better green by letting in a darkened room the light reflected from a paper painted with dark azurite fall on one painted with King's yellow.

§ 9

The strength of the incoming light, even if it is completely white, is very influential concerning the shine of colors because it does not just make them stronger and more vivid, but in most cases also brighter. A wall that is very white can when in the shade appear more and more gray, more so than a less white one or an ash gray one in sunshine. At night the strongest red color as well as the blue one turn totally black and thereby all intermediate steps between them disappear. But the more such steps can be distinguished the brighter the incoming light is. The dark and especially the black colors also suffer some changes when the light is very bright. Because the latter are not so completely black that they no longer reflect any light. To appear appropriately black they only must be exposed to a light of which the reflected portion is very small. Sunlight is not suitable for this purpose, but the light of the sky with white or at least not very dark clouds is. This kind of light with its level of brightness is normally the one for viewing paintings so that they can be seen clearly but do not appear very light.

§ 10

The brightness of the incoming light can increase by very many steps. A white object is in the sun 500,000 times lighter than in the light of the full moon (*Photometry* § 1078) even if sun and moon are equally high in the sky and the air is equally transparent. The light of the star-studded night is so weak that 500,000 stars of the first magnitude would make it barely as bright as the full moon does (*Photometry* § 1152). But that does not mean that there are no steps of clarity. However, the problem is to represent all these steps in painting. In paintings only the color white is what is called light. But between white and black there are barely 30 grades that can be discriminated by the eye. As a result, the painter has to limit himself to certain grades of lightness. In a landscape viewed in sun- or moonlight, neither the sun nor the moon can appear in the painting. The sun itself is 110,000 times brighter than the whitest object illuminated by it (*Photometry* § 777). The same is true for the full moon. If however the sun is seen as a white, pale speck through clouds, so that it is only slightly brighter than the cloud it is easier to represent it in a painting when special circumstances require it. By the way, a painter can let his 30 grades of clarity stand for actual 60, 90 or more grades of clarity of the painted objects. However, his painting will look always more natural if he remains within true limits. But if he extends his 30 grades in a painting to multiple grades in the objects, he cannot do so in a proportionate manner but he must, where a darker objects stands next to a lighter one, increase the difference to approach the difference that exists in the objects themselves. This is particularly necessary when the brighter object is more distant than the darker one. This is a part of what is called, but barely explained by, *chiar-oscuro* or light and dark. It is usually described as having the power to push objects in a painting into the distance, as if linear perspective would not contribute to the effect. It is true, however, that when perspective is neglected, light and dark is the only remaining if very insufficient means. But a view drawn according to perspective shows the distance of each object, even before light, shade, and color are added and without the aid of foregrounds. The outline alone is already sufficient for this. I therefore view chiar-oscuro as just a last resort, to be used when objects have more than 30 grades of clarity that in the painting run from white to black. But it is best if the grades of clarity in the painting and in the objects are the same.

§ 11

By the way, if I describe 30 grades of clarity from black to white I am rather exaggerating than doing the opposite. The painting must be viewed in bright daylight so that each of the steps can be perceived. The less light there is in which the painting is viewed, the fewer steps can be distinguished between white and black. Experiments in this regard are described in *Photometry* § 265-271. On a paper, placed 10 inches away from the light I could distinguish 1/26$^{th}$ of clarity. If the paper was 50 inches away from the light I was only able to distinguish 1/15$^{th}$. These grades are the most distinguishable ones because they represent change in lightness from white to black. But there are considerably fewer grades when the steps pass from red to blue, from blue to black, from red to brown, or from brown to black. These steps also diminish noticeably when the light becomes dimmer. In respect to paintings this has a special effect. If they are painted in daylight the painter can accurately represent any small difference in clarity and express it in the painting. But such a painting must not be exhibited in a dark corner, because there it loses all smaller steps and the capabilities of the painter are seemingly diminished. But the painter must make clear that the expression *Volet haec sub luce videri* ['It will wish to be seen in the light,' Horace, *De arte poetica*, line 363] applies, if not writing it on the painting at least tell the purchasers how they must view it, should they not themselves be smart enough to know this without being told. The colors in the painting are in part changed by a weak light. White appears gray, red tends toward brown, yellowish green toward olive, etc. But this was not the intention of the painter; all the more it is necessary to view the painting in the same light in which it was painted.

§ 12

Just as the painter, so that he can represent the 30 steps from white to black in a manner that they can be distinguished, requires daylight he is also more successful if the objects themselves are lit by daylight. He cannot use completely white paint in so-called night paintings where light from the moon, from a lamp, from candles, or from the fire place illuminates the objects. And that already reduces the number of steps. A landscape that in daylight is exposed to the empty sky can in certain conditions be painted quite well. Only the sun must not be present itself in the painting because its brightness is in a relationship to the brightness of the objects that cannot be represented with paints. The objects themselves are less different in lightness than one might believe. A white object located in the shade of the direct sun light but exposed to the light of the clear sky is slightly less than six times darker compared to direct illumination by the sun (*Photometry* § 1232). If the sun is close to the horizon or its rays fall at a low angle on the object the difference is much less. It is therefore not necessary to paint a wall in the shade in charcoal black; a middle gray is completely adequate. However, quoting *Leonardo da Vinci,* it must be said that if such a painting is only illuminated by daylight the objects will never have the brightness that they have naturally when illuminated by the sun. If illuminated by the complete sky the painting would still be 4, 5 to 6 times darker than if the objects it represents are directly illuminated by the sun. Daylight by itself is therefore also in this case the most suitable illuminant in painting.

## Chapter 3 Differences and relationship of colors where one transitions into the other

§ 13

The modifications of colors just discussed leave their basic nature unchanged. Differences in illuminating light do not change colors themselves but only their shine. If the light is completely white also the resulting shine is in agreement with the true one. This also explains why whenever a color is discussed or named it is taken to be seen in white light. White light, thereby, is the yardstick used when assessing the impact of any colored light. Gloss, vividness, and strength of colors also change them only to the degree that they can change in regard to gloss, vividness, and strength. In all other respects white remains white, black remains black and gray remains gray, etc. The only subject that still needs to be considered is the one stating that white is not black, in other words, a color is not a different one.

§ 14

This difference between colors might be called a **difference of kind**. One would do so even if there were only very few colors from each other in the world different, e.g., white, yellow, red, blue, black. As it is, one color transitions into another and the result of this is that colors are less different from each other **according to kind** than they are **step-wise**. These steps must be counted. They are infinite if one makes the difference from one to the next grade infinitely small. But such infinitely small steps are of no help because we would not be able to distinguish them. I have already noted that placing 30 steps between white and black represents the maximum, and that they have to be viewed in daylight so that they can be distinguished. From white to yellow, from yellow to green, from green to blue, from blue to black the number of steps is noticeably smaller. Accordingly, the infinite number of steps between infinitely small size of difference do not pose a problem. The situation is comparable to the division of the inch in 10 or 12 lines or a foot into half, quarter, or $8^{th}$ of a foot. It is possible to increase the division. But a division that results in parts small enough for practical use, but not infinitely small, is sufficient. There is always a choice of increasing the number of divisions when special purposes call for it.

§ 15

Understanding that colors transition into each other has been used by painters already since a long time to achieve from mixture of a few colors many other colors. In this manner they pull white through all grades of gray into black and they can change any gray, through admixture of another color, so that it transitions to a smaller or larger degree and finally loses itself in it. The main issue was always to reduce the art of mixture to few primary colors with which, if not all but with only a few exceptions, other colors can be mixed. According to a statement by Pliny that is not easy to explain the oldest Greek painters used only four colors: **white, ochre, red**, and **black**. The quote is as follows: *Quatuor coloribus immortalia opera illa fecere, ex albis Melio, ex silaceis Attico, ex rubris Sinopide Pontica, ex nigris Atramento, Apelles, Echion, Melanthius, Nicomachus, clarissimi pictores cum tabulae eorum singulae oppidorum venirent opibus*. [In their immortal works only four colors, Milos white, Attic yellow, red from Sinope on the Black Sea, Atramentum black, were used by Apelles, Echion, Melanthius, and Nicomachus, most famous artists, even a single one of their paintings being immensely expensive.] Pliny added that in his time also purple, Indian earth, dragon- and elephant blood was being used, without achieving good results so that the ancient Greeks, with fewer colors achieved more. In particular, one misses among these oldest four colors blue, still the most difficult to find in good quality. Perhaps atramentum black was sufficiently bluish so that when painted by itself it was black, when mixed with white it appeared bluish. I do not want to concern myself here with the various opinions that have been offered concerning Pliny's statement. Vitruvius describes the colors used in his time in considerable detail but, in his manner, not very clearly. But it is evident that his purpose was not to find the true primary colors required for a theory of color mixture, nor to indicate what they are and, as a result, I will not further concern myself here with this subject.

§ 16

Concerning more recent times I will begin with *Leonardo da Vinci*, the progenitor of the most important recent painters. He was a genius, born for the purpose of resurrecting painting, to note all related details, and to make them known in a fundamental and scientific form. The following excerpt deserves to be translated here.

"The subject of mixture of colors, one with another, being without limit, I want to mention here only a few facts. First, I want to mention a certain number of simple colors, used as a basis for the others, and to mix each of them with the others, first in pairs, and then two with two, three with three, four with four, and so on with all others. To two of such quadruple colors one adds three, to these three other three, and then six, to be continued with mixtures according to all proportions. The name **simple colors** I give to those that are not composed, nor can they be composed from other colors. Even though black and white are not colors because one represents darkness, the other light, one being its destroyer, the other its generator, I do not want to neglect them because in painting they are considered the most noble, because all of painting is composed from light and shade, or from brightness and darkness. Black and white are followed by blue and yellow, also green and lion's yellow, then chestnut brown, *tané* or rather ochre, then plum color or violet-blue, and red, and these are eight colors no longer present in nature. Because I am currently lacking the necessary paper I will describe this matter in detail later in a separate text that will not only be useful but necessary. This description of the subject matter will be in part theory and in part practical experience."

In another place Leonardo mentions only six colors in the following order: **1. White, 2. Yellow. 3. Green, 4. Blue, 5. Red, 6. Black.**

§ 17

Thus, *Leonardo* **only touches** on the subject matter and promises to present it in more detail later, presumably also to make experiments. But the just mentioned introduction has already quite a far reach. Actual testing would have abbreviated the exposition. Of the last mentioned 6 basic colors green, because mixed from yellow and blue, could have been left out. Even black is not necessary, because it can be mixed from blue, yellow, and red. In this manner there remain only four basic colors: **1. White, 2. Yellow, 3. Blue, 4. Red**. But if *Leonardo's* originally mentioned eight colors would be necessary there would nevertheless be a limited list of mixtures because they cannot be presented other than by themselves, in pairs, in threes, ….. all the way to groups of seven, and finally all eight together, and thereby 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = 255 mixtures, if mixed in equal parts. But because they also would need to be mixed in unequal portions the number of mixtures would be much larger. But it is not necessary to make the related calculations because the necessary basic colors are much fewer than eight. It is necessary to mention here that the theory of combinations and their complete calculation has only been discovered more than 100 years after *Leonardo's* death and that therefore *Leonardo* would have had to invent it if he would have wanted to make his proposal complete and on basis of an explicit foundation. The fact that he thought about such color combinations is already very impressive. That he assigns the place of his doctrine to fall between theory and practical application is also appropriate because a painter must first learn how to prepare his colors before he can learn how to apply them. But *Leonardo* thought even further. It was not sufficient for him to mix the colors but he also gave thought to use them at their appropriate place and in their true light. In this manner he speaks in Paragraph 298 (page 72 of the German translation) about a **certain and truthful science** and shows in one example how black, white, and blue need to be mixed in accurately measured and determined portions to appear of necessity natural, when applied to the predetermined place in the painting. The German translator thinks *Leonardo* only joked about this matter. But he was completely serious about it.

§ 18

I am not aware if after *Leonardo* and until the middle of the previous century [17$^{th}$ c.] anybody else made experiments in the scientific mixture of colors. Later **Newton** published his trials with the prism and thereby increased the knowledge about the nature of colors. It became apparent that the rays themselves are colored [!] and objects only have the purpose to reflect them in various mixtures. It also became apparent that the simple colors must be looked for among the prismatic colors, that all mixed together result in white light, and thereby the latter has the most components. Newton imitated the prismatic colors with common colors but found, when mixing them, only a kind of gray that when viewed in sunlight looked as white as a white color in the shade. It was actually a powder mixed stepwise from **auripiment, purple, verdigris**, and **lazur** until it appeared ashgray. The first two together resulted in a pale red, the second two aided in making it gray. It is apparent that auripiment, verdigris, and lazur are already rather light colors aiding in make the mixture lighter.

§ 19

**Newton** divided the prismatic colors into seven classes, red, orange, yellow, green, blue, indigo, and violet and indicates that the widths of these colored ranges, or their distance from the red end, increase like the fractions 1/8, 1/5, 7/8, ½, 2/3, 9/9, 1. The intermediate regions are therefore like the numbers 45, 27, 48, 60, 60, 40, 80. The sum of these numbers is 360, and if one continues counting from there on, 360, 405, 432, 480, 540, 600, 640, 720 are obtained, numbers increasing approximately according to the order of the sounds *a, h, c, d, e, fis, g, a*. If *a* is to be the basic tone, the resulting melody is neither a hard nor a soft. But in this division much is actually accidental. In the prismatic image colors not separated by determined number of clearly separated classes. They transition through imperceptibly small steps from one to the other. On top where true red is located I find a very narrow strip of Florentine lake, or a red that is tinged with blue. Then true red of a high carmine color follows. This transitions into fire red or red lead before it arrives at the middle between red and yellow. Orange can easily be separated into two or three grades. From yellow to green, from green to blue there are clearly also grades that can be distinguished, as this is possible between the three blue and blue-red colors. Whoever wants to find the 12 halftones in colors can, in the prismatic image, find without much difficulty the separation into bands that corresponds to any desired temperature, and can go even further than what is possible in musical chords. But it remains a fact that in the prismatic image the color bands from red toward violet increase in width so that one should take less the sum of their widths but rather the sum of their ratios as their measure, as is the case in music for tones.

§ 20

In the prismatic image there is no black color because all that is in it is bright; therefore also all colors approaching black, such as gray, brown, dark olive, and the color of copper are not present in the spectrum. I pointed two very bright prisms that disperse colors much more strongly than English flint glass at the sun in a manner that the red band of one fell onto the green band of the other to see the appearance of the resulting light mixture. The result was a slightly brownish bitter orange color that might have appeared more like dark brown if I would have let the components fall onto a gray and thereby more shadowy paper. It is easy to comprehend that light alone, and without added shadow cannot appear darkish. However, this kind of shadow is present in the material colors, on the one hand it prevents the formation of pure white in mixtures, but on the other it is the cause of black and the darker mixed colors.

§ 21

From the fact that prismatic colors transition in steps from one to the next it follows that neither Newton's seven nor the greater number mentioned above can be viewed as fundamental colors. All these colors can be represented in a circle, where red borders on violet and thereby shows that red slowly transitions into blue. This increases the difficulty of determining the true fundamental colors and their number, because in the circle there is neither beginning nor end. It can be determined without difficulty that two colors are insufficient, but that **red, blue**, and **yellow** are required. The proof is that all prismatic colors can be mixed from these three. Red and yellow produce all orange colors, yellow and blue the green ones. With blue and red, finally, all indigo, violet, and purple colors can be mixed. But as yet it is not clear which kind of red, yellow, and blue need to be used.

§ 22

In the prismatic image **red** is positioned at one end, but **yellow** and blue have neighboring colors on both sides. At appears that of the latter two the central color needs to be used, but of red the one at the extreme border. Yellow should tend neither toward orange nor toward greenish yellow but must be true yellow. In the same way there should be neither a trace of indigo nor one of green in **blue**. **Red** is determined by itself.

§ 23

In this manner the three fundamental colors are also equally separated from each other. Rather than using Newton's eight intervals I am using the 12 intervals of the complete octave; thus we have the following numbers: 360, 384, 405, 432, 450, 480, 500, 540, 576, 600, 640, 675, 720, whereby the ranges are

From 360 to 405 red
From 405 to 432 orange
From 432 to 480 yellow
From 480 to 540 green
From 540 to 600 blue
From 600 to 640 indigo
From 640 to 720 violet.

These numbers increase in the prismatic image along with the colored regions, beginning with red. But they are not the measure of the perceived difference between the colors. The perceived difference does not depend on the values, but on their ratios, and thereby according to the logarithms of the numbers. To find three equal intervals we have to find two median proportional values between 360 and 720. Not considering the fractions, they are 454 and 571. They thereby fall very near the centers of the yellow and blue ranges, however slightly closer towards the side where yellow as well as blue begin to tend toward green. The difference is not noticeable, but it indicates that in yellow as well as in blue any reddishness must be avoided.

§ 24

Instead of the values mentioned the following can be used that are harmonically related: 360, 384, 405, 432, 450, 480, 540, 600, 640, 675, 720. Without noticeable error the eleven median geometrically proportional values can be used that, when the fractions are disregarded, have the following values: 360, 382, 404, 428, 454, 481, 509, 539, 601, 641, 680, 720. They result in what in music is called comparable temperature. But because the intervals, or the difference between one color and the next, are like in music, not given by these numbers but by their ratios, also here it is appropriate to use their logarithms. If a circle is equally divided into 12 parts, two of these are apportioned to red, one for orange, two for yellow, two for green, two for blue, one for indigo, and two for violet, it will be found that the beginning of red and the centers of the yellow and blue colors will form an equilateral triangle. Because red has 60, orange 30, green 60, and the first half of yellow 30 degrees, together resulting in 120 degrees. The other half of yellow has 30 degrees, green 60, and half of blue 30, thereby in sum again 120 degrees. Finally, the other half of blue has 30 degrees, indigo 30, and violet 60, therefore together also 120 degrees.

§ 25

The separation of the circle into parts is somewhat different from that of Newton. His purpose was to show how the prismatic light rays, if those of any colors are combined by projection on paper result in a colored image and how the color of this image can be determined based on knowledge of the mixture components with a simple method. For this purpose he separated his circle in such a manner that the following sectors resulted: red 1/9, orange 1/16, yellow 1/10, green 1/9, blue 1/10, indigo 1/16, and violet 1/9. These fractions are in the same relationship as the following whole numbers: 80 + 45 + 72 + 80 + 72 + 45 + 80 = 474. The full circle is divided into 474 parts of which the mentioned colors occupy in their order the following parts: 80, 45, 72, 80, 72, 45, 80. However, Newton has done this only to simplify matters in several respects. Because, to bring ratios into a sum it is necessary to add their logarithms and, thereby, the logarithm of 360 would have needed to be subtracted from the logarithms of 405, 432, 480, 540, 600, 640, 720, and the circle would have had to be divided according to what is left over. Here, according to Brigg’s logarithms, are the remaining values and their differences

| Remaining value | Difference |
|---|---|
| 0.0000000 | |
| 0.0511525 | 0.0511525 |
| 0.0791812 | 0.0280287 |
| 0.1249387 | 0.0457575 |

| | |
|---|---|
| 0.1260912 | 0.0511525 |
| 0.2218487 | 0.0457575 |
| 0.2498775 | 0.0280287 |
| 0.3010300 | 0.0511525 |

These differences are not completely in the ratio of the fractions 1/9, 1/16, 1/10, 1/9, 1/10, 1/16, 1/9. But the difference is relatively small and for the purpose for which they are used these fractions are sufficiently accurate.

§ 26

According to the harmonic numbers provided by Newton Father Castel, using his imagination, built a color piano. In his *L'Optique des couleurs* [1740] he spoke against Newton in a tone that was supposed to make a lot of noise. He sees 12 colors in the spectral image, i.e., 1. Carmesin, 2. Red, 3. Seville orange yellow, 4. Golden yellow, 5. Yellow, 6. Olive, 7. Green, 8. Sea green, 9. Blue, 10. *Violant*, 11. Agath, 12. Violet. I want to consider here neither his separation nor his naming. By the way, Castel claims that he is the first who recognized that there are only three basic colors, **Red, Yellow** and **Blue**. In addition, he finds fire red, *Stil de grain* [a yellow lake derived from quercetrin] and true sky blue to be most useful. He does not recognize true yellow that is neither reddish, greenish, nor brownish, because it is treated with contempt by the French and Castel himself views it as a failed white. For his and the French people's satisfaction yellow therefore must be reddish, just as he prefers fire red over true red that has a strong color of blood, presumably because in fire red there is a small amount of yellow present. But we are here not concerned with fashionable colors but with the true, simple basic colors, and for this purpose not a brilliant red looking like the color of fire, but a **strong true** red, not a reddish tinted yellow but a **strong true** yellow, and finally not a light sky blue mixed with much white but a **strong true** blue must be selected. Father Castel also indicates that he can achieve a true **black** by mixing **1/3 part red**, **1/3 part yellow**, and a **full part of blue** and then adjusting the result until the mixture appears black. It is easy to see that also other dark colors can be obtained from such mixtures. But **white** does not appear. White is, of course, not really a color, but light. But in painting it always needs to be used, and in this regard it must be viewed as a fourth basic color, even though it is not one in terms of prismatic colors. The shadows of objects make it a requirement in painting. And even if one would ever find a kind of yellow, red, and blue that in mixture result in a true white it would then be impossible to obtain black or dark colors from their mixture, and therefore there will always be a need for a fourth basic color. To not regard white and black as colors may in certain circumstances be sensible. But here it is only a play with words. Black and white coloration can be achieved just as red, yellow, green, blue and brown can. In a true sense, black and white indicate only the limits of colors. Limits always must be part of a matter that has limits.

§ 27

Approximately at the same time Le Blond issued in London an article titled *Il Colorito*. All I know about it is what was reported in 1751 in **Hamburgisches Magazin** in connection with copper engraver Gautier's art of printing engraved copper plates covered with colors for which he received an exclusive royal patent (identical with *Privilegium exclusivum*). Le Blond contributed the theory by using **red, yellow**, and **blue** as primary colors (with the paper used in copper engravings furnishing **white** as the fourth color), all other colors being derived from them. Among the usual painter's colors he includes 1. Lead white, 2. Naples yellow, 3. white or common ochre, 4. Roman ochre, 5. cinnabar, 6. dark red, 7. English ochre, 8. umber, 9. Cologne earth, 10. ivory black, 11. Berlin blue, 12. mixed earth, 13. stil de grain, 14. Florentine lake.

§ 28

Even though I do not plan to offer here a complete history of the simple basic colors and their mixture, where it is considered systematic, I will in addition to the late famous Göttingen astronomer Mayer, whose color system will below be discussed in considerable detail, mention also the attempts by two other authors. The first is **D. J. Chr. Schäffer's "Proposal for a general color system, or attempt and example of how to determine and name colors useful for the general public"** This proposal was published in 1769 and delivers on one page the common colors as they exist without further mixture in nature, or produced artificially. On the other page there are samples of various red colors, obtained from mixtures of the first group. It is evident from this example that in addition to the main colors many more others should also be developed and perhaps they will. The other work, or at least its beginnings, was published in this same year of 1772 under the title **"Attempt at a color order system developed by Ignaz Schiffermüller of the Jesuit Society in the Imperial Theresian College."** The author does not just consider water colors but any kind of painting and dyeing and is asking actual artists for useful contributions towards the completion of the project. **Father Schiffermüller** knows Mayer's color triangle, at least by name and general content, published in 1758 in issue 147 of the **Göttingische Anzeigen**, reprinted because of its significance in Vol. 4 of the old **Bibliothek der schönen Wissenschaften**. It seems apparent that Schiffermüller, just as many others, was not able to fully comprehend this triangle because, as is apparent to me, he prefers the color circle of Father Castel of the Jesuit Society while rearranging the latter's colors to some degree. He separates the color circle into twelve parts and colors them in the following order **1. Blue, 2. Sea green, 3. Green, 4. Olivegreen, 5. Yellow, 6. Orange, 7. Fire red, 8. Red, 9. Carmesin red, 10. Violet red, 11. Violet blue, 12. Fire blue** so that they flow from one into the other, such that carmesin, blue, and yellow form an equilateral triangle and thereby are equidistant. **Father Schiffermüller** indeed takes yellow, red, and blue as the true basic colors. The second copper engraving contains samples of three kinds of blue, each transitioning in steps upwards toward white and downwards toward black.

## Chapter 4  Mayer's color triangles

§ 29

The only information I have on this subject is the just mentioned article in issue 147 of *Göttingische Anzeigen* 1758 that I am repeating here as it has been printed in the *Bibliothek der schönen Wissenschaften***.**

"The treatise of Professor Mayer, read on Nov. 18 during the public meeting of the Society of Sciences, is a proposal for the measurement of colors with the help of mixture. In the large variety and number of colors there are not more than three that deserve to be called simple or basic colors because they cannot be produced by mixture of other colors while, at the same time, they can be used to mix all the other colors regardless of what their names are. These three colors are red, yellow, and blue; they are most distinct in the rainbow, but even livelier in the beam of sunlight separated with help of a prism, even though they are accompanied by other mixed or secondary colors. Some, according to Newton, like to also consider these secondary colors as basic colors and in this manner end up with seven, i. e., red, orange, yellow, green, blue, indigo, violet. But because these folks do not clearly explain what their definition of basic color is, their opionion cannot be taken as a counter-argument. In reality one finds in the sun beam separated by prism in addition to the three basic colors all the secondary colors that are generated from the mixture of two basic colors. Orange, as well as the various forms of golden yellow, saffron yellow, color of fire, etc., located in the prismatic beam between red and yellow are nothing else but mixtures of the two. Equally, all versions of green appearing between yellow and blue are mixtures of those two, as violet, indigo, purple, rose-color, and similar colors with which the prismatic beam is bordered on both sides are just mixtures of red and blue. Mayer conjectures that even in the light beam originally there are no more than three basic colors; at least they are the only ones visible, without the mentioned mixed colors, if one views a single black spot on a white background in appropriate distance through a prism. Colors generated from mixture of all three basic colors are not seen in the rainbow or in the prismatic beam. Their number is much larger than those just mentioned, even though only a few of them have well-known names. Among these are all kinds of browns and greens. The number of all possible colors is infinitely large because the difference between colors depends on the variation in the ratios according to which they are mixed from the basic colors and these ratios can be changed in infinitely many different ways. But all resulting colors cannot be equally well differentiated by our eyes. If, for example, only 1/30th of blue is mixed into yellow a green should result but it would be hardly distinguishable from yellow itself. Only those colors are clearly distinguishable where the ratio can be expressed with moderately large numbers. The number 12 can be considered here as applicable when relationships are selected, just as in architecture and music, higher values are rarely chosen. If the names of the basic colors red, yellow, and blue are abbreviated with their initial letters as **r, g**,[FN] and **b** and indicates the number of parts of each used in a mixture

FN The initial of the German word gelb (yellow) is maintained here so that later figures can be copied from the original.

of all three, so that a neighboring color is obtained, by superscripted numbers, the number of neighboring colors falling between always two basic colors is eleven. These mixtures can be expressed as $\mathbf{r}^{1}\mathbf{g}^{11}$, $\mathbf{r}^{2}\mathbf{g}^{10}$, $\mathbf{r}^{3}\mathbf{g}^{9}$, etc. Their total number is 33. The colors mixed from all three basic colors can be identified as $\mathbf{r}^{1}\mathbf{g}^{10}\mathbf{b}^{1}$, $\mathbf{r}^{2}\mathbf{g}^{9}\mathbf{b}^{1}$, $\mathbf{r}^{1}\mathbf{g}^{9}\mathbf{b}^{2}$, $\mathbf{r}^{3}\mathbf{g}^{8}\mathbf{b}^{1}$, etc. and if the total of the superscripted numbers is 12 they will always be distinguishable. Mayer named the superscripted numbers *Partienten* [indices] to distinguish them from the exponents of the mathematicians. The total of colors mixed from three basic colors is 55. If the above mentioned 33 and the 3 basic colors themselves are included the total of colors that can be distinguished is 91. Because this number is a trigonal number, with each side having 13 members, colors can be represented in total in an equilateral triangle with 91 subdivisions so that the three basic colors are located in the apices of the triangle, those that are mixed from two basic colors along the sides, and those mixed from all three in the interior. The mixed colors are placed the closer to a basic color the more the more of it is present in the mixture. The result is a kind of color measuring tool in which every existing color can be identified according to the amounts of basic colors in the mixture, nearly in the same way that on a test stone the composition of mixtures of gold and silver can be determined. Using this measuring tool Mayer has investigated nearly all colored earths and artificial pigments used by painters in oil painting and determined for each the related indices that show the ratios in which they are mixed from the basic colors, or at least can be mixed. Only the following few are given here as examples: orpiment and king's yellow $\mathbf{g}^{12}$, yellow ochre $\mathbf{r}^{2}\mathbf{g}^{10}$, auripigment $\mathbf{r}^{4}\mathbf{g}^{8}$, dark ochre $\mathbf{r}^{3}\mathbf{g}^{8}\mathbf{b}^{1}$, umber $\mathbf{r}^{3}\mathbf{g}^{6}\mathbf{b}^{3}$, burnt umber $\mathbf{r}^{4}\mathbf{g}^{4}\mathbf{b}^{4}$, chrysocolla $\mathbf{g}^{4}\mathbf{b}^{8}$, minium $\mathbf{r}^{9}\mathbf{g}^{3}$, Cologne earth $\mathbf{r}^{4}\mathbf{g}^{3}\mathbf{b}^{5}$, English red $\mathbf{r}^{6}\mathbf{g}^{2}\mathbf{b}^{4}$, ivory black $\mathbf{r}^{3}\mathbf{g}^{2}\mathbf{b}^{7}$, cinnabar $\mathbf{r}^{12}$, Florentine lake $\mathbf{r}^{8}\mathbf{b}^{4}$, Berlin blue $\mathbf{r}^{1}\mathbf{b}^{11}$, azurite $\mathbf{b}^{12}$. Mayer showed with a few examples how any desired color can easily be mixed from two or maximally three basic or intermediate colors, a fact that will prove very helpful in painting. For example, color $\mathbf{r}^{2}\mathbf{g}^{1}\mathbf{b}^{9}$, a kind of iron color can be most easily mixed from equal parts of ivory black and Berlin blue. It is to be understood that all that has been said so far relates to colors that have perfect strength and tend neither toward paleness nor toward darkness. But each of them can, without a need to change its name, have different grades of paleness, achieved by mixing them with white. The number of these colors, the completely pale or white color itself included, is 364. The same number applies to colors tending toward darkness, the most extreme of which is black, obtained from any color when it totally lacks light or whiteness. Mayer also provided a convenient numbering system for these weakened as well as the darkened colors which expresses their nature. He concluded with a review of the richness and inexhaustibility of the art of painting that, as can be concluded from the above, has 819 distinguishable colors at hand which in different compositions make it possible to produce infinitely many works." [See the image of Mayer's basis triangle, as published in 1758 in Appendix 2.]

§ 30

When I read this report in the newspaper in 1758 I naturally assumed that Mayer's presentation would appear soon after in *Göttingische Commentarien*, and that the author himself would make an attempt to produce and place into the hands of enthusiasts at least one of his triangles. At that time I only drew from the information given as above the examples of the various mixtures and constructed the triangle according to Mayer's definition. Because in his examples **Mayer** gave cinnabar the designation **r**$^{12}$, king's yellow **g**$^{12}$, and azurite **b**$^{12}$, I concluded that he used these as his basic colors **red, yellow**, and **blue** and I mentioned this also in #1189 of Photometry. **Mayer** then passed away and it does not appear that his works, purchased by the government of Hannover, will be published any time soon as **Mayer** had hoped. As I suspected that few people would be able to sufficiently comprehend the concept of **Mayer**'s triangles from the above news article in *Göttingische Anzeigen* I used the occasion of my treatise *Sur la partie photometrique de l'art du peintre* (Mem. de l'Acad. R. de Berlin 1768) [On the photometric aspects of the painter's art] where there was an appropriate occasion, to demonstrate **Mayer**'s triangle, to report his ideas, and to point out, in particular, that the proportions of basic colors required for each color require further in-depth investigation. Also Prof. **Sulzer** found **Mayer**'s color triangles important enough to describe them in his **Allgemeine Theorie der schönen Künste** [Johann Georg Sulzer, General theory of the arts, 2 vols. Leipzig: Weidmann 1771-74] and to add several important comments. In the beginning, Mayer planned only to consider mixed colors as colors, to determine the number of possible but clearly distinguishable mixtures, and to place them in an ordering system. Viewed in this manner, if they are just viewed as colors that are easily distinguishable, one from the next, he came up with 9 x 91 or 819 kinds. This number has an arbitrary component in setting the number of steps between one basic color and the next at 12. In bright light and especially between red and yellow or blue and yellow there might be more than 12 detectable difference steps; in reduced light there are fewer. Between red and blue 12 steps are even in bright light not sufficiently detectable; at night red and blue, unless there is white mixed in, is difficult to distinguish or not distinguishable at all, even though yellow is still clearly recognizable. What in addition Sulzer was looking for relates to the differences that can be recognized in **juice, warm**, **transparent, glazed**, or **full colors applied in the deepest shadow** and thereby also in cases of **clever pointillist painting**. These differences do not change the colors themselves but rather their action in the eye. They are not the result of simple mixture of colors but require other additions or applications of craft, whereby one enhances that which the colors lack, and thereby they are applicable to any color. A ***Saftfarbe*** [colorant prepared from the juice of a plant] is by itself transparent and is used in glazing over **earth colors**. Full colors in the deepest shadow have to be gauged by the light falling on them and usually must be shown in greater intensity, as already mentioned earlier. This is due to the fact that the painter does not have the means to represent more than 30 grades of lightness and he cannot represent the loss in clarity that is very strong in nature even more strongly in the painting beyond a relative reduction to maximally 30 grades. But in reality painters often exaggerate such strong declines in lightness without any need to do so. Sulzer concludes by saying "the complete treatment of coloration in painting is a task suitable for a painting academy such as the one in Paris that accepts as members the most capable and experienced masters of the art of painting." Coloration and its complete representation in a theory has aspects beyond the number of possible colors and their modification in regard to gloss, strength, vividness, etc. The application of color with the painter's brush, the technique of transitioning one color into another, the determination of grades of gloss, of reflection, of strength, of vividness, of lightness of objects, the relative limitation of the resulting grades to those that are possible in painting and rarely reach 30. All that must also be considered if coloration is to be reproduced as accurately as distance, apparent form, position, and size are by the application of perspective. To achieve this a new *Leonardo da Vinci* is required, possessing the necessary knowledge of optics, physics, and mathematics, applying himself with comparable attention to the subject, collecting related materials, and combining everything into a system.

§ 31

Assuming with Mayer that even in prismatic colors there are only three basic colors, **red**, **yellow**, and **blue**, one nevertheless has to concede that each ray with a particular degree of refrangibility [wavelength] must have very special properties, such that some of the red rays must have the same wavelength as some of the yellow rays because, so as to generate the orange color in the prismatic image, they must coincide. But this is not possible without having the same degree of refrangibility. In addition, some yellow rays must have the same degree of refrangibility as some blue rays because otherwise there cannot be anything green in the prismatic image. Also, there must be some red rays that have the same degree of refrangibility as the most extreme blue rays, else neither indigo nor the strongly reddish violet color of the spectral image would appear. That such differences in degree of refrangibility of identically named color rays can exist and partly must exist becomes apparent when one views through a prism a black paper strip on white paper. One border only shows the blue and violet, the other only the red and yellow colors. In this manner one observes two colored images out of which the Newtonian image, with inclusion of green, is composed if it is reflected at a sufficient distance, or if a white paper strip on black paper is viewed in appropriate distance from the eye through the prism. If the distance is insufficient, the center of the white paper is seen as white and the two colored images are separated. These matters are described in more detail in **Beguelin**'s treatise '*Remarques et observations sur les couleurs prismatiques*' (Mem. de l'Acad. Roy. de Berlin 1764) where there is a detailed description of the as yet little researched facts of the mixture of colored rays in the prismatic image. **Mayer**'s conjecture of only three kinds of rays is quite likely factual because, in fact, any other prismatic color can be mixed from the yellow, red and blue ones, but not vice versa. But, because it is necessary to give the red, yellow and blue rays in part the same degree of refrangibility, doubt arises that, regardless of this, they can appear red, yellow, and blue. It is necessary to find a means to separate them, regardless of their identical degree of refrangibility to determine the separate action of each on the eye. As is known, **Newton** only separated those that can be separated because of different degree of refrangibility.

§ 32

The mentioned article in *Göttingische Anzeigen* does not indicate how **Mayer** came up with the idea of distributing the mixtures from the three basic colors in a triangle. Already when he was very young he presented in his **Mathematical atlas** published in Augsburg [1745, see Appendix A] a very simple example of color mixture. At that time he used five basic colors A white, E yellow, I red, O blue, and V black. These he mixed in equal parts in combinations of two and ordered them as follows.

AE EI IO OV
AI EO IV
AO EV
AV

Above these he placed his five basic colors and below also a mixture of all five. In the same manner it would have been possible to represent mixtures of 6 and more colors in the form of a triangle. The 6$^{th}$ color would have added the additional hypotenuse AY, EY, IY, OY, VY. Mixtures of five colors in triples could have only been represented on a surface in the following form

aei aeo aeu
aio aiu
aou

---

eio eiu
eou

---

iou

Mayer did not do this and neither did he represent the mixtures of colors in groups of four that could have been done only as follows

aeio, aeiu, aiou, eiou.

In all of these the mixtures would only have been in equal parts. It is therefore obvious that in case of unequal parts there would have been many more combinations and their ordering would not have been intuitive.

§ 33

It seems obvious that later **Mayer** realized these difficulties. He therefore began by deleting from his five basic colors **white** and **black** and to keep only **red, yellow**, and **blue**. Mixed in equal parts they would have resulted in a very simple figure

r g b

---

rg rb
gb

---

rgb

Along these lines there was room to expand on his thinking and to consider also unequal mixtures.

§ 34

To explain this I will first use only two colors, for example red r and blue b. They can be mixed as follows

| | | |
|---|---|---|
| 1. | Alone | r, b |
| 2. | In equal parts | $r^2$, $r^1b^1$, $b^2$ |
| 3. | In thirds | r3, $r^2b^1$, $r^1b^2$, $b^3$ |
| 4. | In quarters | $r^4$, $r^3b^1$, $r^2b^2$, $r^1b^3$, $b^4$ |
| 5. | In fifths | $r^5$, $r^4b^1$, $r^3b^2$, $r^2b^3$, $r^1b^4$, $b^5$ |
| 6. | In sixths | $r^6$, $r^5b^1$, $r^4b^2$, $r^3b^3$, $r^2b^4$, $r^1b^5$, $b^6$ |
| | etc. | |

Here the numbers indicate how many parts are used of each color. For example, $r^4b^2$ indicates that there are four parts red and 2 parts blue to be mixed. This could also have been expressed as

$$4r + 2b.$$

But it is evident that Mayer preferred the first more abbreviated version.

§ 35

If the third color, yellow g, is added, one, two, three, and more parts of yellow will need to be mixed in with each of the above mixtures. This results in the following cases.

1. Basic colors alone

| | |
|---|---|
| r | |
| b | g |

2. Half and half mixture

| | | |
|---|---|---|
| $r^2$ | | |
| r b | r g | |
| $b^2$ | bg | $g^2$ |

3. One-third mixtures

| | | | |
|---|---|---|---|
| $r^3$ | | | |
| $r^2b$ | $r^2g$ | | |
| r $b^2$ | r b g | r $g^2$ | |
| $b^3$ | $b^2g$ | b $g^2$ | $g^2$ |

4. Quarter mixtures

| | | | | |
|---|---|---|---|---|
| $r^4$ | | | | |
| $r^3b$ | $r^3g$ | | | |
| $r^2b^2$ | $r^2bg$ | $r^2g^2$ | | |
| $rb^3$ | $rb^2g$ | $rbg^2$ | $rg^3$ | |
| $b^4$ | $b^3g$ | $b^2g^2$ | $bg^3$ | $g^4$ |

5. Fifth mixtures

| | | | | | |
|---|---|---|---|---|---|
| $r^5$ | | | | | |
| $r^4b$ | $r^4g$ | | | | |
| $r^3b^2$ | $r^3bg$ | $r^3g^2$ | | | |
| $r^2b^3$ | $r^2b^2g$ | $r^2bg^2$ | $r^2g^3$ | | |
| $rb^4$ | $rb^3g$ | $rb^2g^2$ | $rbg^3$ | $rg^4$ | |
| $b^5$ | $b^4g$ | $b^3g^2$ | $b^2g^3$ | $bg^4$ | $g^5$ |

In this manner one can continue until any desired number of parts is achieved. **Mayer** chose twelfth and doing so results in

| | | | | | |
|---|---|---|---|---|---|
| $r^{12}$ | | | | | |
| $r^{11}b^1$ | $r^{11}g$ | | | | |
| $r^{10}b^2$ | $r^{10}bg$ | $r^{10}g^2$ | | | |
| $r^9b^3$ | $r^9b^2g$ | $r^9bg^2$ | $r^9g^3$ | | |
| $r^8b^4$ | $r^8b^3g$ | $r^8b^2g^2$ | $r^8bg^3$ | $r^8g^4$ | |
| $r^7b^5$ | $r^7b^4g$ | $r^7b^3g^2$ | $r^7b^2g^3$ | $r^7bg^4$ | $r^7g^5$ |
| ꝛc. | | | | | |

§ 36

The number of squares in these triangles grows as the sum of natural numbers 1, 2, 3, 4, 5, 6, 7 etc. Accordingly, they are represented by the trigonal numbers 1, 3, 6, 10, 15, 21, 28 etc. **Mayer** calculated according to twelfth. The number of squares in his triangle is found by adding the arithmetic progression from 1 to 13, or (1+13)/2 multiplied by 13. The result is 91 colors, obtained by **Mayer** using twelfths of **red, yellow**, and **blue**.

§ 37

Next **Mayer** adds to these colors on the one hand **white**, on the other **black**. It has been mentioned that the number of colors mixed with white is 364, as is the number of colors mixed with black. As a result, the total number of those mixed with white and black plus those mixed from **red, yellow**, and **blue** = 364 + 364 + 91 = 819. Several things need to be mentioned in regard to this sum. It turns out that 364 is equal to 4 times 91. This could be interpreted that the 91 twelve-part portions are just always mixed with 1 to 4 parts white, or only by 4 steps toward white and the same toward black. But this is probably not applicable. Because it also would have been possible to mix the 13 colors $r^{12}$, $r^{11}b^{1}$, $r^{10}b^{2}$, $r^{9}b^{3}$, . . . . . . $b^{12}$ each in 12 steps toward yellow. But the resulting visual differences would in the end have been too small. But it is certainly true that with few steps one can see how every color is diluted toward white. But we can interpret the number 364 in another way that appears more appropriate.

§ 38

Toward this goal we want to investigate the case where four colors are mixed together. It is easy to see that the combinations become more complicated. In the case of two colors we have one, only linear, dimension. In case of three colors we obtain a triangle and thereby two dimensions. So we likely will have to expect three dimensions involving not just lengths or widths, but length, width, and height. As a matter of fact we do not obtain in this case a single triangle but multiple ones resulting in the form of pyramids, as I will show presently. There are mixtures of 4 colors, r, b, g, w,

1. Each individually

| | |
|---|---|
| w | |
| r | |
| b | g |

2. In identical parts

| | | |
|---|---|---|
| $w^2$ | | |
| $w^1r^1$ | | |
| $w^1b^1$ | $w^1g^1$ | |
| $r^2$ | | |
| $r^1b^1$ | $r^1g^1$ | |
| $b^2$ | $b^1g^1$ | $g^2$ |

3. In thirds

| | | | |
|---|---|---|---|
| $w^3$ | | | |
| $w^2r^1$ | | | |
| $w^2b^1$ | $w^2g^1$ | | |
| $w^1r^2$ | | | |
| $w^1r^1b^1$ | $w^1r^1g^1$ | | |
| $w^1b^2$ | $w^1b^1g^1$ | $w^1g^2$ | |
| $r^3$ | | | |
| $r^2b^1$ | $r^2g^1$ | | |
| $r^1b^2$ | $r^1b^1g^1$ | $r^1g^2$ | |
| $b^3$ | $b^2g^1$ | $bg^2$ | $g^3$ |

etc.

Here the number of squares increases in any kind of mixture according to the sums of the trigonal numbers, therefore like the pyramidal numbers 1, 4, 10, 20, 35, 56, 84, 120, 165, 220, 286, 364, 455, etc. In this series we immediately notice the number 364 that in Mayer's 12-part mixture shows the squares actually mixed with white. If the 91 squares are mixed only from red, yellow, and blue we obtain 455 squares. And the number 455 is actually the pyramidal number associated with the trigonal number 91 and the root 13.

§ 39

But it is not quite evident why **Mayer** also wants to make mixtures with **black** as **red**, **yellow**, and **blue** already produce a true ivory black and any color tending toward black. **Black**, thereby, is not required for other mixtures. But if it is decided that black should be taken as a basic color one would proceed according to the same scheme and the sum of the sum of the squares would be equal to the sum of the earlier mentioned pyramidal numbers 1, 4, 10, 25, 35, …… 455, thereby according to the order 1, 5, 15, 35, 70, 126, 210, 330, 495, 715, 1001, 1365, 1820, etc. so that with five basic colors according to 1/12 part mixtures the result would be 1820 colors. They would have to be ordered into a row of 13 pyramids, consisting of 1, 2, 3, 4, 5 etc. triangular layers and thereby have 1, 4, 10, 20, 35 etc. squares. This series could still be drawn well in a perspectival manner. But it is not necessary because we can limit ourselves to 4 basic colors, **white, red, yellow**, and **blue**.

§ 40

Because Mayer's triangles and a related treatise have not been published I will not go into much detail in regard to issues in part already mentioned earlier. It is not evident from the report in *Göttingische Anzeigen* how Mayer determined the amounts of color to be used in each mixture. It is evident that each portion in the mixture should have an identical effect. But it does not follow from this that a portion of red, or yellow, or blue must have the same volume or weight. But this must be accurately investigated in advance. The determination of colors to be used in mixtures to obtain all others has its own difficulties. I commented on these already earlier when discussing Castel's color circle, with **Castel** clearly deviating from true **red** and **yellow** and also used a too light **blue**. Mayer's $r^{12}$, $b^{12}$, $g^{12}$ are cinnabar, azurite, and king's yellow. I have two kinds of azurite where one is darker than the other, but even then is too light. King's yellow is too reddish. Cinnabar is redder than **Castel**'s fire red, but nevertheless not red enough. With an addition of blue Mayer was able to match the hue of Florentine lake. But between that and cinnabar there are also carmine and blood red that cannot be matched with cinnabar and blue. In addition, a reddish yellow mixed with blue results in a green with a more or less brownish-green olive color and thereby is not a true green. It is not at all easy to make a good selection of basic colors. Painter's colors are not as pure as spectral ones but it is necessary to select those that approach perfection as closely as the nature of the colors available to us allows. But one should not forget that not all colors can be mixed equally well. Many that by themselves work well show only in mixture that they consist of components of different kind that become more apparent in mixtures and thereby ruin the mixture. Often this becomes apparent in water colors because those colors that have heavier particles sink right away to the bottom while the others remain on top so that the mixture must be constantly stirred when being applied. The result is that they can never be applied well.

## Chapter 5  Calau's wax

§ 41

In my view **Mayer**'s color triangle, because of its systematic mixture and ordering of colors, deserves full attention. Coloration should advance at the same pace as perspective. *Leonardo da Vinci* considered both. During his time perspective obtained, if not all necessary simplification, completeness, and sufficient usefulness and was advanced to geometric science level and certainty. Coloration, as much as Leonardo with theoretical approaches and examples pointed in the direction of a **certain and true science** as the real goal of its completion, nevertheless remained too much in experimentation and tricks of the trade to be learned in extended study. Each painter acted in an individual **manner** and rarely according to **nature itself**, proceeding in the matter of colors, light, and shade in the same way as those that have not learned perspective proceed in the matter of distance, position, and apparent size.

§ 42

However, I saw **Mayer**'s triangle only as an attempting to determine how far one can proceed in color mixture using **cinnabar, king's yellow**, and **azurite**. That they are less than perfect as basic colors I have already mentioned. But what the limitations are could not be judged in greater detail without having an actual painted triangle. After the death of Mayer it was no longer possible to assume that one would be able to learn more about his triangle or to see it further improved. When considering an imitation I wasn't much encouraged by the earlier mentioned difficulties. And it was not easy to explain it to practiced painters only with words. I nevertheless made an attempt by painting a triangle, however only at the 1/6th portion level, as well or poorly as I could. But this triangle was more a hieroglyphic image of the matter than the matter itself. It nevertheless turned out to be more useful than only words to explain to a painter what might be achieved if the project would be properly undertaken. The painter I was looking for had to have the following attributes: 1. To see his own extensive practical ability not as the extreme limit of all possible ability but to continually search for new paths and to be convinced that beyond his current range of vision there is still much hidden and that from a new and partly higher point of view he might be able to see new and more distant pathways. 2. One that knows the miscibility of every color and its steps, having much practical experience and if possible an open mind to grasp the proposal and its goals, as well as able to apply his practical capability to an appropriate degree toward what the mathematically valid theory, or to speak with Leonardo, the certain and true science, requires. 3. He should be a good colorist, able to judge immediately which of the basic colors being considered results in the largest possible number of mixed colors. 4. And finally he should, based on his own need to know, be willing to produce the various required test samples.

§ 43

This is a quite considerable list of requirements, all to be met by one candidate. I did not want to search the earth to find one. And even if I would have wanted to do so I could have started the search for good reasons in Berlin. A conversation with Mr. **Meil** [Johann Wilhem Meil, 1733-1805, painter and engraver, book illustrator] on the subject of the arts was an opportunity to raise the question. That this opportunity was appropriate only needs to be explained to newcomers that as yet do not know what they should think when hearing the name **Meil**. Mr. **Meil** introduced me to the court painter **Calau** and he in turn to his rediscovered **Punic**, or as he calls it **eleodoric**, [FN] wax and to the multiple and very different kinds of using it in painting. What I was immediately interested to hear concerns its property to be soluble in water like a gum resin, that it unites different colors very well once it is dried and keeps them fresh-looking as they did when applied. This immediately resolved one of my important concerns. I showed my triangle to Mr. **Calau**, even though it looked very hieroglyphic and was pleased to see that **Calau** did on it not see the colors as I applied them but those that, beautified with his wax, should have been applied. He commented: "Here we only have cinnabar, but there are three red colors and I want to try all three. I like this problem." On the next day he handed to me three triangles where he used the three different reds 1. Carmine, 2. Florentine lake, 3. Cinnabar, and where he also changed yellows and blues. Our preference fell on **carmine**, **Berlin blue**, and **gamboge**.

But before I continue with this matter I want to return to a closer consideration of the issue of the wax and inform the reader in more detail about it.

§ 44

Wax originates in plants. In some plants, particularly plants growing in the Americas, one finds it, even though in a reduced level of quality, already in a form that can be extracted with warm water. In most plants, and especially in flowers, it must first be sucked up by bees and digested, so that it increases in viscosity. Like resin, it has liquidity, viscosity, and extensibility. It disperses in water of barely 56˚ R [158 ˚F] a temperature at which alcohol does not begin to boil yet. According to Cartheuser's experiments [Johann Friederich Cartheuser, 1704-1777] warm alcohol extracts a limited amount of oil of beautiful golden color that gives wax its yellow color. If afterwards more alcohol is added and brought to a boil the remaining wax turns into a kind of white butter. However, approximately 1/5$^{th}$ remains unchanged and contains blackish earth particles. Cartheuser , from whom I am quoting these experiments (*Fundamenta materia medicae*, vol. 1, section IV, paragraph 8) says that neither the oil nor the butter of wax can be seen as true dissolution of wax but must rather be a crude and violent separation of parts.

§ 45

How, in the bodies of bees, wax is separated from honey and if this can be achieved artificially, is beyond our discussion. What is clear about the composition of honey is that in addition to watery and resinous slimy parts there are many oily and solid parts. Wax is essentially free of watery and resinous slimy parts but has in addition to oily, fatty parts further components specific to it. I placed a small amount of yellow wax onto a rock and aimed light from a lens onto it of a kind that can result in lead melting. The wax melted, dissolved more and more and only a few minor parts appeared to be burnt black. After cooling down, the melted wax had a brown color and because the layer was thin it remained transparent and continued to be capable of melting.

§ 46

I think it useful to have begun this discussion with a consideration of the common wax produced by bees. Its components have their source in the world of plants, but they usually are composed differently than those found in bee's wax. The latter has oily parts but also parts that appear to be the components that make wax into wax. The oily parts only provide the color and contribute toward viscosity and stickiness. It appears that they are responsible for the insolubility of wax in water or alcohol but that they make possible miscibility with resin, turpentine, and oils, etc.

§ 47

This raises the question if the earlier mentioned wax particles cannot be obtained in a form that is mostly or completely free from oil. With the oil present it is easy to understand that with addition of an acid a kind of soap is obtained. [?] But this does not mean separating the true wax particles from the oily parts but just the addition of acid to both. Mr. **Calau** told me that the matter must be addressed in a completely different way and that it is easier and less time consuming to find, just as the bees do, the actual wax particles in the plants and their juices. Perhaps there they are mixed with much less oil and not nearly as closely connected. This seems to be a conjecture based on good reasons. Mr. **Calau** claims it is not a conjecture and offers the wax he produced as proof. It needs to be more closely investigated to come to a conclusion.

§ 48

However, it is not easy to carry out such an investigation. Calau's wax is a wax and it is not a wax. I am not playing with words and so I have to explain myself in more detail. One thing is very clear: his wax is in many respects hugely different from the one produced by bees. But the difference is only due to the fact that it is free of the oily and buttery parts which are intimate parts of bee's wax. That is why it can be completely dissolved in water just like gum resin and in alcohol like varnish. But if the water or alcohol evaporates it remains unchanged, covering and sticking to the bottom of the glass as if one would have melted common white wax in it after which it had solidified again. In both of them there is white color, transparency, viscosity, extensibility and also in regard to smell they are only different in degree, with Calau's wax smelling less. It dissolves in water just like gum resin und combines well with it. Spirits of nitre dissolves it without problem and after it evaporates the wax remains unchanged. I placed a small piece of Calau's wax behind the mentioned lens but kept the lens quite close to reduce the heat. The surface began to be glossy, but the wax kept its form and size, however it appeared to become looser in consistency. I increased the heat on the wax. It became more liquid and slowly slid off. With heat increased further it dissolved and began to smoke. Some parts remained solid and between them there was a brownish oil or thick oily juice. In the center of the applied heat the harder solid parts turned to a kind of blackish-brown coal or pitch. The liquid parts also turned more brownish but did neither expand further nor smoked any longer. After cooling off it solidified and was still capable of dissolving in water so that, except for its change in color, it continued to be **Calau**'s wax. Mr. **Calau** also uses it with great success in place of oil of spike or other agents, not known to me, to apply colors on fayence and porcelain. With equal success he draws with his wax colors on paper maché and creates a varnish by exposing it to the heat of fire. His wax dissolves in oil and reduces the degree to which oil in oil painting attacks the canvas. I need to add that when placed in water solid particles of the wax drop to the bottom but soon turn milk-white and if stirred they give water a milk-white color. When shaken, the water produces bubbles, but without colors on them such as can be seen on soap bubbles.

§ 49

It is evident that **Calau**'s wax is a special substance. It considerably differs from common wax. Nevertheless, common wax is the only natural agent with which one can compare it because it has the properties of viscosity, extensibility, softening when held in the hand, a yellowish color, turning white in water, bleachability, the smell of common wax and, like the latter, it differs from fat, oil, soap, resins, etc. It thereby meets all the tests for wax and only begins to show differences when tested in more detail by hydrostatic and chemical means. Even in chemical tests there is doubt because it is not possible to separate in bee's wax the initially mentioned oily parts from the true wax parts so that one could compare the latter directly against **Calau**'s wax. But in my opinion this is what would need to be possible for a chemical comparison of **Calau**'s wax. If that is possible some differences one might find will not allow for an immediate decision in the matter. It is not claimed that **Calau** has found a way to keep the waxy particles in his wax completely separated from any other materials. Equally, a chemist cannot be expected to fully separate bee's wax into pure wax and other materials. It suffices that the true wax particles in either product are demonstrably identical. **Calau** has the right to call his wax **wax** and to make known all its advantages, as he has already done in several short announcements and he can be expected to continue doing so.

§ 50

What has been mentioned so far concerns **Calau**'s wax alone. Mr. **Calau** is not satisfied knowing that it is wax and that it offers several different applications. He also claims that he rediscovered the **Punic** wax used in many different ways by the ancient Greek painters who did not know oil painting. Only in a few limited cases such as painting on boards did they mix the wax with oil, and it served them much better than oil alone. Several years ago Count **Caylus** discussed this kind of wax painting of the ancients, or rather he brought back to general attention what is known about it, hoping to revitalize the technique, if possible. [FN] This gave **Calau** the opportunity to duplicate with his wax all the various methods used by the ancients as can be deduced from the vague reports by **Pliny** and others. Calau is capable of achieving encaustic painting, burning-in of colors, painting on walls, the linear style of painting, etc., in a manner that can help clarify the reported sources.

But I will not concern myself further with this matter as this is not the appropriate place. The proof I consider the most convincing of all those offered and what seems to have a certain historical likelihood is the news that **Calau,** without having looked for them, found the first traces pointing toward his wax in Russia. When painting their saints early Russian painters, since ancient times, used certain plant juices in the preparation of the paints. **Calau** based his wax on this information, but making various improvements. It needs to be considered that the Russian church derived from the

FN Anne-Claude-Philippe de Tubières, Comte de Caylus (1673-1709), author of *Mémoire sur la peinture à l'encaustique et sur la peinture à la cire*, Paris: Pissot, 1755.

Greek church and that the first Russian saints very likely were painted by Greek painters. Thus one can conclude that a certain amount of information concerning Punic wax and the art of how to use it in painting has found its way into Russia from Greece. In the 10$^{th}$ century Russian warring ventures led them far as Constantinople. Other in part earlier historical circumstances might provide additional incentive to search in Russia for remaining, if modified, usage of ancient Greek methods. But there is also the question if Punic wax is not older than the Greeks. It is also known by the name **Pontian** wax. **Pliny** describes its preparation with the following words: *Punica cera fit hoc modo. Ventilatur sub divo saepius cera fulva. Deinde servet in aqua marina ex alto petita addito nitro. Inde lingulis havriunt florem, id est candidissima quaeque, transfundunt in vas, quod exiguum frigidae habeat. Et rursus marina docoquunt separatim: deinde vas ipsum refrigerant. Et cum haec ter fecere, iuncea crate sub dio siccant sole lunaque: haec enim candorem facit, sol siccat: et ne liquefaciat protegunt tenui linteo. Candidissima vero sit post isolationem etiamnum recocta.* To proceed in this fashion is approximately what we today call **wax bleaching**. But it is not apparent how it can be said of this wax: *pingere ceris*. It is an expression that seems to indicate a lost art of the ancients, frequently mentioned by the great Göttingen historian **Gesner** [Johann Matthias Gesner 1691-1761, classical scholar and educator] as something quite peculiar. The quote from *Seneca* "*Pictor colores, quos ad reddendam similitudinem multos variosque ante se posuit, celerrime denotat, et inter ceram opusque facili vultu et manu commeat*." appears to show that they first covered the surface to be painted with wax and then push the colors into it, proceeding quite rapidly. But this is at most just one method of many of painting with wax. *Varro* describes a similar method: "*Pausias et caeteri pictores eiusdem generis loculatas habent arculas, ubi discolores sunt cerae*."

So in this case, already the colors themselves are wax colors or colors prepared with wax. By the way, the difference between the two terms is significant regarding the question if **wax is in the colors** or **colors in wax**, in other words if the largest and most important part of the mixture is color or wax. **Calau** doesn't worry about this distinction because he can easily achieve either situation. If he combines the colors with wax he uses less of the latter than the amount of gummi resin that normally would be used and in case of earth colors he uses color and wax in equal parts. If he works the colors into wax he covers the surface with wax and draws with a crayon with a particular ability and in a way that appears to clarify the linear art of drawing by the ancients.

## Chapter 6 Selection of the basic colors

§ 51

In my first triangle, in a hieroglyphic format (§ 41, 43), I used **cinnabar**, **gamboge**, and **litmus** as the basic colors because I did not want to undergo the pain of having to grind azurite, Berlin blue, or king's yellow and I only wanted to obtain a general picture of mixture and organization of colors to be able to show it to a painter. In **litmus** there is quite a bit of red, not producing a true green when mixed in 1 part with 6 parts of

**gamboge**, but it showed barely a trace of brownish, greenish olive color. **Cinnabar** and **gamboge** produced quite minium and orange colors of fair quality. Carmine or blood color was not obtained. Finally, **cinnabar** and **litmus** produced only copper colors, in part reddish and in part tending toward black-brown-blue, rather than producing true violet and purple colors as should be obtained from mixture of **red** and **blue.** This was not the result of much red in **litmus** because this would only have increased the redness in **cinnabar**. Rather, I had to draw the conclusion that there must be a significant portion of yellow in **cinnabar** because the generation of copper colors requires the presence of all three basic colors.

§ 52

I already mentioned that after I showed him my triangle **Calau** produced on the following day three other triangles, with different reds, yellows, and blues. In one of them he used **cinnabar**, **king's yellow**, and **Berlin blue** as basic colors, using his wax in making the mixtures. All colors looked good in regard to their mixture. Minium, orange and green colors were all present. But there was no carmine, blood color, violet, purple, lake, or rose color. Again, the conclusion was that in cinnabar must contain a noticeable amount of yellow.

§ 53

This was even more obvious in comparing Calau's second triangle, based on **carmine**, **gamboge**, and **Berlin blue**. **Carmine** and **gamboge** in mixture produced in order blood color, cinnabar, minium, auripigmentum, golden yellow, orange yellow, etc. **Berlin blue** and yellow appropriately produced all green colors that fall between them. Finally, **carmine** and **Berlin blue** produced all lake, violet, and purple color. It should be noted here that there can be very noticeable differences between carmine from different sources. Sometimes it resembles cinnabar, sometimes more Florentine lake, and in both cases the mixtures turn out more or less unsatisfactory. I had carmine from three different sources available. The least expensive one closely resembled cinnabar and looked fine by itself, but in mixtures it resulted mostly in impure, dead, muddy, or flat colors. The second one had a burning red appearance and was closer to true red than to lake and was better in mixtures with **gamboge** than those with **Berlin blue**. The third was darker and it resembled lake more than true red, but was nevertheless of burning appearance and pure. Mixtures with blue, using **Berlin blue** were slightly better than mixtures with gamboge, but both kinds were of quite good quality. Also in case of **Berlin blue** there are products with different degrees of purity, fineness, lightness and darkness. The darker and thereby stronger one turns lighter when applied in weaker dilution and thereby passes through several grades. Nevertheless, a version that is fine and pure is preferable. Gamboge is a juice color, turning brownish-reddish when applied in high concentration. As a result, it must be applied at a certain level of dilution if its appearance is to be pure yellow without a trace of redness.

§ 54

In his third triangle Mr. **Calau** used **Florentine lake** as the basic red color, together with **gamboge** and **Berlin blue**. The resulting purple colors looked very good. However, the carmine and blood colors as well as cinnabar and fire colors were mostly or completely absent. From these preliminary experiments it became obvious that for the purpose of water colors the best choices are **carmine**, **gamboge**, and **Berlin blue**. They do not lack in beauty and can easily be mixed. **Gamboge** is actually a juice color. **Carmine** and **Berlin blue** produce opacity, but much less so than earth colors. They can be considered to fall half way between juice and earth colors. All three are suitable for drawing and they can be applied as diluted or heavy as desired, each one by itself as well as all three in mixtures. It is known that the joint use of juice and earth colors, as has been practiced in the past and was often necessary, is frequently not a good solution when illuminating small landscapes, flowers, etc. The latter cover up copper engraving lines, the former are transparent. When painting with them according to one's taste, earth colors result in coarse spots, often of a deathly color, while juice colors have greater vividness, frequently being used as a glaze for the deathly earth colors. In case of the mentioned three basic colors such difficulties are nearly absent. **Calau's** wax mixed with the binder gives them life and strength of any desired degree.

§ 55

By the way, I leave it open for discussion if in future colors will be found that will more closely approximate the true prismatic basic colors. Perhaps it will be possible to produce the mentioned **carmine** and **Berlin blue** even more pure and perfect and closer to prismatic purity. What I mentioned earlier will always be the required proof. **Carmine** mixed with gamboge must produce pure prismatic hues between red and yellow, without any appearance of brownishness. **Berlin blue** mixed with **gamboge** must pass through all steps of greenness, without degrading into brown-olive. And, finally, carmine mixed with **Berlin blue** must produce all violet, indigo, and purple colors of prismatic purity and good mixture, without tending toward brown. I know that it is not easy for the magicians of material separation to always achieve this goal and to the same degree of perfection. But as yet they have not discovered all the tools of their art. And so there will be further improvements. In the meantime the mentioned tests will serve painters and anybody else working with the indicated colors to purchase the best **carmine** and **Berlin blue,** leaving the poorer preforming ones behind.

## Chapter 7 Determination of the strength of the basic colors

§ 56

**Calau** mixed the colors for his earlier mentioned three triangles just as I did those for my own hieroglyphic image of such a triangle: only with the brush. For a preliminary test this can be considered satisfactory. The purpose of those tests was mainly to deter mine the miscibility of the colors and to what degree the wax keeps the mixtures homogeneous and makes their colors more vivid and in part also stronger. In this regard the mixtures from red to yellow, from yellow to blue, and from blue to red, whereby always only two colors were mixed, posed no difficulties because one could always start with different amounts or add to them until the desired number of intermediate grades was obtained, assuming that the differences between them are not too strongly uneven. This was more difficult when mixing three colors in a manner that the result could be considered to be correct for its place in the triangle. But Mr. **Calau** was too much of a colorist not to have been able to at least imagine the required intermediate colors. For every brownish color tending toward yellow, green, blue, purple, violet, cinnabar, and minium he easily found its appropriate location. However, intermediate nuances, the place for which was not easily predictable, and in particular the location of fully black colors that appeared among the mixtures, was not at all clear, even though it was obvious that the latter must be located much closer to blue than to red or yellow. Together with **Calau** I also made an attempt at 1/16$^{th}$ mixtures, whereby all mixtures from the three basic colors can be found by sequential halving. In case of the binary mixtures this worked out quite well. In one situation, the middle color by halving between cinnabar color and a slightly bluish green was to be found or to be guessed. This was not easy to achieve just by mixing with the brush. Mr. **Calau** also said that he preferred to start with green and proceed through darker olive colors toward brown and then from the reddish brown all the way to red itself and to evaluate how he could produce the required number of intermediate colors. There was some success, but the outcome always was that without having the first one for comparison and attempting to produce 10 or more by themselves the triangles did not resemble each other in regard to the appearance and order of colors, in each one several nuances were clearly wrong, some colors differing too much from their neighbors, others not enough. Initially, **Calau** thought the problem to be that all colors were applied in equal strength and that by appropriate intermixture of white the situation could be improved, so as to have the advantage that master colorists obtain from light and shade. But after a while I was able to convince him that the cause of the lack of order was due to the fact that the amounts to be mixed must be determined much more accurately and in a manner applicable anywhere. Already from the start I had proposed **weighing** of the portions that belong to each mixture, and I did not want to and neither was able to deviate from that because I wanted to have mathematical accuracy in the steps of each mixture, a situation much easier to achieve with a gold balance than with the judgment based on observation. **Calau**, like any master colorist, did not want anything to do with a balance. However, he agreed to an experiment. Good reasons for such an approach were not difficult to find. The most straightforward and surest way to repeat the mixture of a defined color, as often as desired, is the one where each portion of color required for the mixture is defined by weight as a portion of the total weight of the mixture, regardless of the units of weight.

§ 57

The key question now was how to proportion the basic colors according to weight. This would be very simple if all of them were of equal **strength**. But they are far from it. Since its invention **carmine** has not only been admired for its beauty, closely approaching that of the spectrally purest red, but also for the fact that it is very strong and a unit weight of it provides more coloration than several units of weight of other colors. This situation is of considerable influence in mixture with other colors and thereby increases the difficulty of determining the required weight ratios. I already noted that there can be considerable differences between carmines from two sources. This means that the ratios determined for one carmine must be changed accordingly for a carmine from another source. I will therefore describe my experiments in such a manner that one can apply the results for any other kind of carmine and make the corresponding changes.

§ 58

That I had three kinds of carmine at my disposal I have already mentioned in the previous chapter, where the only issue concerned the determination of its quality. I discarded the one of poor quality completely because it resulted in mixtures of a dirty appearance. I also had two kinds of Berlin blue, a lighter and a darker one. So I used at the beginning the **highly intensive carmine**, the lighter **Berlin blue**, and **gamboge**. The latter two were finely ground by Mr. **Calau** who was present at these trials so that they, just like the carmine, could be weighed in form of a powder and mixed in a dry state before they were ground again on the rubbing stone with water, binder, and wax. This was done in preparation for determining the strength.

§ 59

Subsequently I weighed very accurately half a gran of carmine and half a gran of gamboge. [FN] The mixture, applied heavily, began to have the color of blood. As a result I still was far away from reaching the middle point between carmine red and gamboge yellow, a color that falls between cinnabar and minium. I continued to add gamboge in increments of half a gran. Each new combination was painted on regal paper and the result indicated that only after reaching the 10$^{th}$ half gran of gamboge was the yellow color in the mixture as strong as the red one, thus 1 half-gran of carmine being in mixture equally strong as 10 half-gran of gamboge.

§ 60

I now weighed ½ gran of Berlin blue and, by sequentially adding **gamboge** in increments

FN "Gran" was the designation for a unit of weight in a system before the metric system was introduced. In the apothecary weight scale of the 18$^{th}$ century: 1 Gran = 62.5 milligram.

of ½ gran, I found that three half-gran resulted in a mixture that was still too blue, while 4 half-gran resulted in a color already tending toward yellow. From this I concluded that 3 ½ half-gran was the amount coming closest to the neutral green, neither tending toward blue nor toward yellow or, at most, a trace toward yellow. As a result, such a green requires 2 gran of Berlin blue and nearly 7 gran of gamboge.

§ 61

Finally, I also weighed ½ gran of **carmine** and added **Berlin blue** in increments of ½ gran and found that three half-gran still resulted in a slightly reddish mixture while 4 half-gran produced a mixture that appeared strongly bluish, so that 1 gran of carmine and a little over 3 gran of Berlin blue resulted in the middle color between red and blue.

§ 62

My experiments lasted for an afternoon of weighing, mixing, and painting. I made the judgments toward evening, in already reduced light, but with the colors still looking fresh. The next day, in strong midday light, I noticed that, especially in case of green and red-blue intermediate mixtures no changes were required. However, in case of the red-yellow mixtures the result was less certain, so that if a change ever would be necessary it would be for that last mixture. For this reason I made the following comparison for all three mixtures.

§ 63

For each mixture the resulting ratio was different. The three ratios are

| Carmine | Berlin blue | Gamboge |
|---|---|---|
| 1 | | 10 |
| 1 | 3+ | |
| | 2 | 7 |

As a result we have 5 different numbers. They need to be reduced to three numbers that represent the **degree of strength** of the three basic colors in a general manner. The question arises if in two of the given relations the third one is already implicitly determined. This should be the case if each basic color maintains its strength in each mixture. To find if this applies in our case a test is required. Green consists of 2 gran blue and 7 gran yellow. Blue is therefore 3 ½ times stronger than yellow. Red is three times stronger than blue. From this we can conclude that red is 3 times 3 ½ or 10 ½ times stronger than yellow. The practical result is in agreement with this as accurately as one can expect. As a result of these comparisons I found that the **degrees of weakness** can be taken to be as follows:

| | |
|---|---|
| Carmine | 1 |
| Berlin blue | 3 |

| | |
|---|---|
| Gamboge | 10. |

These are the numbers that we attempted to find. I repeat one more time that they apply to the **more intensely red carmine** and the lighter **Berlin blue**.

§ 64

I then made the same experiments again using the **darker carmine** and the **darker Berlin blue** in addition to **gamboge**. The result was somewhat different because all comparisons produced the following degrees of weakness:

| | |
|---|---|
| Carmine | 2 |
| Berlin blue | 3 |
| Gamboge | 12, |

They indicate that in mixtures 2 gran of **carmine** are equally strong as 3 gran of **Berlin blue** and 12 grans of **gamboge**, or that gamboge is 6 times weaker than carmine and 4 times weaker than Berlin blue, the latter being 1 ½ times weaker than carmine. Because gamboge was identical in both sets of experiments it follows that while the darker carmine has lower lightness it also has less **strength** than the earlier used lighter one. But the darker Berlin blue also was the stronger product. Because earlier we found the lighter one to be only 3 times as strong as gamboge but here the darker one is 4 times as strong. At the same time I determined that the darker one, particularly in regard to blueness, is preferable because it can be mixed more easily.

§ 65

What I earlier termed measure of **weakness** of colors is based on the fact that the more of a color is required to achieve a defined mixture the **weaker** it is. Accordingly, the degree of weakness in the just reported tests is 2 in case of carmine, three in case of Berlin blue, and 12 in case of gamboge. If interested in the degree of strength, they are found to be in the inverse ratio of the same numbers ½ , 1/3, 1/12, or in case of carmine 6, of Berlin blue 4, of gamboge 1. It is easy to see that all these numbers are only to be taken as relative, without an absolute measure as their basis. There is no independent unit applicable to any color. The only possibility is to place one particular color with which all other colors are easily and well mixable as the basis and to compare all other colors against it. In the mentioned experiments gamboge, separately mixed with both carmine and Berlin blue, can be used as the common standard. In the former case the degrees of **weakness** were 1, 3, 10, and thereby the degrees of **strength** 1, 1/3, and 1/10, or 30, 10, 3. In the latter tests they are 6, 4, 1, or multiplied times 3, 18, 12, 3. This results in the following degrees of **strength** for the five kinds of color

| | |
|---|---|
| Gamboge | 3 |
| Berlin blue, light | 10 |
| Berlin blue, dark | 12 |
| Carmine, dark | 18 |
| Carmine, stronger | 30. |

My preference is for degrees of weakness because they are directly determined by the weight if the mean mixture is to be determined by experiment and weighing of the colors that are to be mixed. In this way it is easy to comprehend that when in our trials, for example, 2 gran carmine, 3 gran Berlin blue, 12 gran gamboge have the same effect and thereby the same **strength**, the strength itself must relate in each gran, or in another unit of mass or weight, as the fractions ½, 1/3, 1/12.

§ 66

The degrees of strength and weakness of colors are in many ways similar to hydrostatic weight or lightness of solids. For example, it can be said that when carmine and Berlin blue are mixed in a manner that the mixture neither tends toward red nor toward blue but results in the intermediate between red and blue, these two colors are in the mixture in balance, or they act with the same strength on the visual nerves. The eye senses any degree of strength or weakness, and in this manner there is truly something static or dynamic happening.

§ 67

The degree of strength of any desired mixture can be found just as in hydrostatics the total weight of mixed materials can be found from each weight of the mixed materials. Let's assume, for example, as in these experiments, that the strength of carmine is 18, that of Berlin blue 12, and that of gamboge 3. A mixture is made of 4 gran carmine, 6 gran Berlin blue, and 9 gran gamboge. As a result, in this mixture there are

| | | |
|---|---|---|
| | 4 x 18 = 72 | degrees of red |
| | 6 x 12 = 72 | degrees of blue |
| | 9 x 3 = 27 | degrees of yellow |
| and thereby in | 19 gran 171 | degrees of strength. |

If the sum of grades, 171, is divided by the weight of the mixture, 19 gran, one obtains for each gran of mixture 9 gran of strength of color.

## Chapter 8 Calculation of mixtures of basic colors in differing grades

§ 68

I have already noted in my explanation of Mayer's color triangles that **Mayer** indicated the portions of basic colors to be used in any desired mixture, but he did not mention how the basic colors were to be tested and, especially, what the meaning of 'proportions' was to be. This neglect has been eliminated with the two previous chapters. The choice of basic colors fell on **carmine**, **Berlin blue**, and **gamboge**, when mixed with water, binder, and Calau's wax. For the purpose of the following discussion the darker carmine and Berlin blue, mentioned in the previous chapte,r were selected. Their strength in mixture has been determined in advance and was found to be

2 gran carmine
3 gran Berlin blue
12 gran gamboge,

having the same strength in mixtures. It is easy to see that if larger stocks of mixtures are to be produced for later use or for sale, in place of 2, 3, 12 gran the same number of larger units such as *Quentchen*, *Lot, Unze*, etc. can be used [larger units of weight of the time, where 1 Unze = 2 Lot = 8 Quentchen = 32 Gran]. For this reason I will be using the more general term: **part** or **portion**, but with the understanding that 2 parts carmine, 3 parts Berlin blue, and 12 parts gamboge have the same strength in mixtures but each must contribute equal weight in the mixtures, specifically after Berlin blue and gamboge have already been ground to fine powders. This is important because in a test connected with this issue 4 of 200 gran Berlin blue were lost during grinding and of 320 gran gamboge 9 were lost as a result of dusting, sticking to material, and other reasons.

§ 69

If, for example, the mixture named according to Mayer's method $r^3b^2g^3$ is to be produced with the mentioned three basic colors the meaning is that the strength or degree of strength of the red must be 3, of the blue 2, and of the yellow 3. By weight we require for 1 degree of strength 2 parts carmine, 3 parts Berlin blue, and 12 parts gamboge. Accordingly

Three degrees of strength of 2 parts of red results in 6 parts by weight
Two degrees of strength of 3 parts of blue results in 6 parts by weight
Three degrees of strength of 12 parts of yellow results <u>in 36 parts</u> by weight
In total 48 parts.

If the total mixture is to weigh 48 gran one would use for the desired mixture $r^3b^2g^3$ 6 gran carmine, 6 gran Berlin blue, and 36 gran gamboge. If one wants to produce only 10 gran then, according to the stated rule, one would have to say that 48 is related to 6, 6, 36 as 10 is related to 1 ¼, 1 ¼, 7 ½ gran. So one would use the same amounts of carmine and Berlin blue, that is 5/4 gran, but of gamboge 7 ½ gran.

§ 70

The process is comparable in case of any other mixture, and the calculation can be abbreviated. For example, for $r^6b^4g^1$:

$r^6$: 6 times 2 = 12
$b^4$: 4 times 3 = 12
$g^1$: 1 times 12 = 12

so that for this mixture equal amounts by weight of the three basic colors are used. For $r^4b^3g^2$ the result is

$r^4$: 4 times 2 = 8
$b^3$: 3 times 3 = 9
$g^2$: 12 times 2 <u>= 24</u>
41

So that if the weight of the mixture is to be 41, the required weight of carmine is 8, of Berlin blue 9, and of gamboge 24 parts.

§ 71

But if the starting point is weight and one mixes 8 gran carmine, 12 gran Berlin blue, and 20 gran gamboge it can be calculated what the degree of strength of each basic color in this mixture is, because

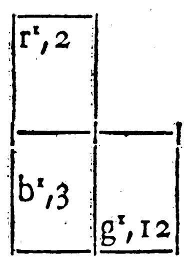
8 divided by 2 = 4; therefore $r^4$
12 divided by 3 = 4; therefore $b^4$
20 divided by 12 = 1 2/3; therefore $g^{5:3}$

the mixture therefore being $r^4b^4g^{5:3}$ or expressed differently $r^{12}b^{12}g^5$, so that of 12 + 12 + 5 = 29 degrees of strength of the mixture red has 12, blue 12, and gamboge 5.

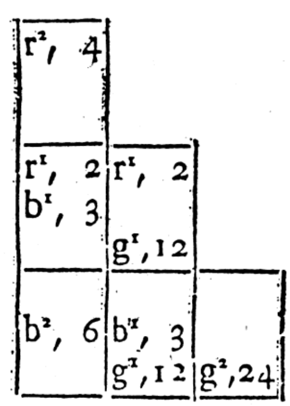
§ 72

If for the total mixture a certain number of degrees of strength is set, the possible combinations can be calculated in whole numbers in a manner already previously shown (§ 35) with examples where the complete mixture had 1, 2, 3, 4, 5 etc. degrees of strength and where for red, blue, and yellow also the number of grades of strength was given. If, accordingly, in place of grade of strength, or together with it, the weight is to be determined it can be done as follows.

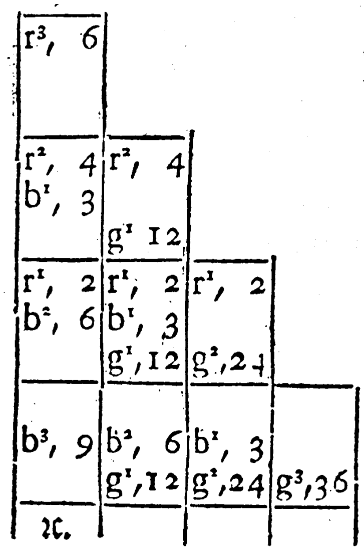
1. Basic colors alone




2. In halves



3. In thirds




§ 73

One can in this manner continue to determine the mixtures in ¼, 1/5, 1/6, etc. parts according to weight by calculating for each part of red 2 parts in weight, for each part of blue 3 in weight, and for each part yellow 12 parts in weight. If it is desired that each mixture has the same weight it can be accomplished easily by applying the rule mentioned earlier. For example, for $r^1b^1g^1$ we have the weight parts 2, 3, 12, with a total of 17. So as to obtain the given total weight, we need to use 2/17 carmine, 3/17 Berlin blue, and 12/17 gamboge. To have mixtures of 10 or 20 gran an easy solution is to weigh a piece of cardboard and then to calculate how large in area 2/17, 3/17, 12/17 of the cards of 10 or 20 gran are, and thereby find the required weight for red, blue, and yellow. [FN] For example, a Tarock game card weighs 41 gran and the desired color mixture weight is 20 gran. Along the lengths of the card 20/41 are cut off and this rectangle will weigh 20 gran. The piece is divided along its length in 8/17 and 9/17. The resulting pieces can easily be divided into 2 times 4 and 3 times 3 parts, adding up to 20 gran divided into 1/17 parts with which it then is possible to weigh the portions 2/17, 3/17, 12/17 of red, blue, and yellow.

§ 74

My calculations are only offered as examples, because they are not applicable to any grade of carmine or Berlin blue without changes, but rather specifically to those used in the later experiments of the previous chapter where 2 gran carmine, 3 gran Berlin blue,

FN It should be kept in mind that at the time of writing this text the only means of weighing such small amounts were gold or apothecary balances, where the desired weight was placed in one pan in form of coins or other reference weights, to be balanced by the weight of the material. Few unit weight types were available. Lambert offers a way to deal with this problem.

and 10 gran gamboge were found to have the same degree of strength. But the calculation can be easily conceived in form of a general formula. Define *m* gran carmine, *n* gran Berlin blue, and *p* gran gamboge to have the same degree of strength. For each mixture $r^{\mu}b^{\nu}g^{\pi}$ the following weights are required

$\mu$ *m* parts carmine
$\nu$ *n* parts Berlin blue
$\pi$ *p* parts gamboge.

The mixture is calculated according to $1/(\mu + \mu + \pi)$ parts and its total weight is $\mu m + \nu n + \pi p$ parts. If it is to weigh A gran, the amounts to be weighed dry and mixed are as follows

Gran carmine: $\dfrac{A\mu m}{\mu m + \nu n + \pi p}$

Gran Berlin blue: $\dfrac{A\nu n}{\mu m + \nu n + \pi p}$

Gran gamboge: $\dfrac{A\pi p}{\mu m + \nu n + \pi p}$.

## Chapter 9 The color pyramid

§ 75

Neither in the experiments nor in the calculations of the previous three chapters did I consider white as a fourth basic color. The plan was to make use of water colors without making use of earth color pigments. It would not have worked well anyway because there is no earth color comparable to carmine, while cinnabar tends distinctly toward yellow, orpiment toward red, azurite is too light. We do not have a white juice color. In case of **carmine**, **Berlin blue**, and **gamboge, paper** itself, if it is quite white, can serve as **white** color if the colors are thinly, to various degrees, painted on it. This thin application does not only work in case of each of the mentioned basic colors but also in any mixture, so that each can be applied just like an India ink.

§ 76

Based on these observation it would only be a matter of applying colors that are to have a certain degree of whiteness correspondingly more diluted, in agreement with the amount of white one would have to add. The amount of water necessary for the dilution would have to be weighed, or it would have to be made certain by trial and error that the degree of lightness one gives the color is what is required. But these are details that can largely be neglected. White does not change anything of the color except making it lighter. If it can be made visible how it transitions toward lightness, most of what we want to gain with addition of water has been achieved. It suffices to show how colors transition toward white, or how they become lighter, in a few noticeably different steps, even though the steps might not be as uniformly different from each other as is required when viewing mixtures of other colors. Because red, green, blue, brown, and gray transition with increasing darkness into black it is easy to comprehend that there are multiple mixtures that, when applied intensely, appear to be black. But when they are applied more diluted not all of them will look gray, but some reddish, bluish, greenish, or brownish and they can therefore not be used in weaker applications without having different appearance.

§ 77

What has just been said concerning black does not apply to all other colors, Yellow, for example, requires few steps to transition to white, and these steps can be imagined without difficulty. In case of red and blue there are more steps, but it is sufficient to show only a few to be able to envision all of them. Those falling between the prepared ones can be easily imagined. In each case the distance of these colors from white represents one unit, of which intermediate mixtures are fractions.

§ 78

I will now proceed with a description of how the color pyramid, delivered in this publication, was produced. The publisher would have preferred an octave format of the book and indeed in that format it would have been easier to carry it in a pocket. The pyramid itself would have required folio format to be larger, more handsome, and perhaps also more detailed. But it would not have been prudent to fold the paper on which the image is presented, because it could not be unfolded in such a manner that all colors would be illuminated identically. Having a separate page for each triangle would also not have been a good idea, because the system of colors is better shown in total, so that it can be viewed in **one** glance on **one** page.

§ 79

As a result the question arose how to design the pyramid in perspective in form of an open triangular box divided into sections on a quart sheet, and thereby obtain as complete a picture as the size of the space and the symmetry to be maintained in the division into sections allow. The result is now available for viewing. The vanishing point is 3 1/3 inches [*Zoll*] above the top of the pyramid and so that the squares appear to have right angles the distance between the eyes and the pyramid must be 18 inches.

The three lower compartments are twice the distance from each other compared to the upper ones. Their distance from the top is related as the numbers 10, 8, 6, 5, 4, 3, 2, 1. In the same ratio are also the front lengths or hypothenuses of the triangles. As a result the front corners of squares are in all triangles equally distant. The other sides become narrower according to perspective.

§ 80

Each triangle represents a totality. In the lowest there are 45 squares, each painted in a color appropriate to its position. The meaning of this is that all mixtures derived from red, blue, and yellow are separated into 45 classes or compartments, ordered in such a way that the colors transition from one to neighboring ones in all directions and each color is bordered by those that deviate least from it, with the difference between one color and the next immediately recognizable. As a result there is, for example, no red found in the outer row running from yellow to blue. The following parallel rows contain, in sequence 1, 2, 3, 4, 5, 6, 7, 8 portions of red. Comparably, in the outer row running from red to blue there is no yellow. The following rows parallel to it have in sequence 1, 2, 3 … 8 portions of yellow. Finally, in the front row changing from yellow to red there is no blue. The rows parallel to it have in sequence blueness of 1, 2, 3, …8 portions of blue. So that they can be compared more easily I am showing all mixtures below.

| $b^8$ |

| $b^7g^1$ | $b^7r^1$ |

| $b^6g^2$ | $b^6g^1r^1$ | $b^6r^2$ |

| $b^5g^3$ | $b^5g^2r^1$ | $b^5g^1r^2$ | $b^5r^3$ |

| $b^4g^4$ | $b^4g^3r^1$ | $b^4g^2r^2$ | $b^4g^1r^3$ | $b^4r^4$ |

| $b^3g^5$ | $b^3g^4r^1$ | $b^3g^3r^2$ | $b^3g^2r^3$ | $b^3g^1r^4$ | $b^3r^5$ |

| $b^2g^6$ | $b^2g^5r^1$ | $b^2g^4r^2$ | $b^2g^3r^3$ | $b^2g^2r^4$ | $b^2g^1r^5$ | $b^2r^6$ |

| $b^1g^7$ | $b^1g^6r^1$ | $b^1g^5r^2$ | $b^1g^4r^3$ | $b^1g^3r^4$ | $b^1g^2r^5$ | $b^1g^1r^6$ | $b^1r^7$ |

| $g^8$ | $g^7r^1$ | $g^6r^2$ | $g^5r^3$ | $g^4r^4$ | $g^3r^5$ | $g^2r^6$ | $g^1r^7$ | $r^8$ |

§ 81

In the second triangle there are 28 colors, with the mixtures calculated in 1/6 parts. These colors are applied slightly lighter and the mixtures are somewhat different from those in the lowest triangle. The three corner colors and those mixed in equal pairs of two are only different in their degree of lightness. All others deviate more or less from those in the lowest triangle, however comparatively very little. The arrangement is the same and each mixture can be recognized from the position on which it has been applied. The following figure helps to make comparison easier:

| | | | | | | |
|---|---|---|---|---|---|---|
| $b^6$ | | | | | | |
| $b^5g^1$ | $b^5r^1$ | | | | | |
| $b^4g^2$ | $b^4g^1r^1$ | $b^4r^2$ | | | | |
| $b^3g^3$ | $b^3g^2r^1$ | $b^3g^1r^2$ | $b^3r^3$ | | | |
| $b^2g^4$ | $b^2g^3r^1$ | $b^2g^2r^2$ | $b^2g^1r^3$ | $b^2r^4$ | | |
| $b^1g^5$ | $b^1g^4r^1$ | $b^1g^3r^2$ | $b^1g^2r^3$ | $b^1g^1r^4$ | $b^1r^5$ | |
| $g^6$ | $g^5r^1$ | $g^4r^2$ | $g^3r^3$ | $g^2r^4$ | $g^1r^5$ | $r^6$ |

If to the 6 portions of color in this triangle 2 portions of white are added the result is again 8 portions, the same number as used in the lowest triangle where however no white is present. The implication of this is that the second triangle is ¼ lighter than the lowest. If we achieved this accurately when applying the colors is not really important. It suffices if the second triangle is noticeably lighter than the first and it becomes obvious how the darker color transitions into the lighter one. A more accurate composition would have been time consuming and expensive. But it was not just left to the persons who had the task of painting the triangles; rather Mr. Calau took them to his place where they worked under his constant supervision. The colors as weighed by me were ground with Calau's wax and in part also with binder and I did not fail, especially in the beginning and then also after completion, to inspect the painted squares to determine if the degrees of lightness needed some adjustment so that at least the more noticeable inequalities would be minimized. A thorough and completely equal density of color in each triangle was on the one hand not obtainable without much additional effort, on the other not really necessary as our plan was to only show a few noticeable steps toward white.

§ 82

In the third triangle there are 15 colors, applied even more lightly. All of them are also found in the lowest triangle when every 2nd, fourth, 6th row is disregarded. The reason is that in the lowest triangle colors are mixed in 1/8 parts, but in the third one in ¼ parts. Based on this it is easy to understand that, for example, $r^4g^4$, $r^2g^2b^4$ etc. in the lowest triangle and $r^2g^2$, $r^1g^1b^2$etc. in the third triangle represent the same mixtures. For a simpler comparison I am showing here the mixture applicable to each square of the third triangle.

| | | | | |
|---|---|---|---|---|
| $b^4$ | | | | |
| $b^3g^1$ | $b^3r^1$ | | | |
| $b^2g^2$ | $b^2g^1r^1$ | $b^2r^2$ | | |
| $b^1g^3$ | $b^1g^2r^1$ | $b^1g^1r^2$ | $b^1r^3$ | |
| $g^4$ | $g^3r^1$ | $g^2r^2$ | $g^1r^3$ | $r^4$ |

If the portions shown here are doubled one finds the 15 mixtures that are the same in the third triangle as in the first but are applied lighter. To the four portions of the basic colors 4 portions of white should have been added, thereby having been applied twice as light. But just having applied them lighter must suffice.

§ 83

In the fourth triangle there are 10 colors, calculated in 1/3 parts. Because in the second triangle they have been calculated in 1/6 parts, the colors of the fourth triangle already appear in the second triangle, but have been applied lighter in the fourth. The mixtures are as follows:

$$b^3$$
$$b^2g^1 \mid b^2r^1$$
$$b^1g^2 \mid b^1g^1r^1 \mid b^1r^2$$
$$g^3 \mid g^2r^1 \mid g^1r^2 \mid r^3$$

§ 84

The fifth triangle contains 6 colors, the three basic colors and their intermediate mixtures, calculated in half parts to which one should add 6 parts white so that they appear twice as light as those in the third and four times as light as those in the first or lowest triangle. But also here it suffices that they have been applied very light.

$$b^2$$
$$b^1g^1 \mid b^1r^1$$
$$g^2 \mid g^1r^1 \mid r^2$$

§ 85

Finally, in the sixth triangle there are only the three basic colors, applied even lighter, that is

$$b^1$$
$$g^1 \mid r^1$$

§ 86

The seventh triangle contains a single square and, so that it represents the color white, it has been left completely white. Above it, just for purposes of symmetry, there is another tiny compartment that does not represent anything.

§ 87

The mixtures, here presented for each triangle, are already sufficient to determine every color in regard to the basic colors from which it is composed. But so that, for purposes of comparison, each one can be easily identified I believe it is most convenient to number them in a way that where a mixture appears in several triangles it always has the same number. This system can be found in the included page [see last page] where I placed the six triangles in sequence so that they can be easily compared with the painted ones. The sum of these numbers is 67, or if the white square is included 68. The same number of mixtures is shown in the triangles of the pyramid. In total there are 108 squares and, if each one is counted as a color, the same number of mixtures. This number is sufficient if what is desired are steps noticeably different from each other. Anybody who wants to expand on this can prepare triangles according to the earlier presented instructions where the steps are distinctly smaller. The color pyramid shown here is entirely sufficient as a very useful example.

## Chapter 10 General remarks concerning the color pyramid

§ 88

It is now my task to indicate how the color pyramid presented here is to be viewed in regard to colors and their application. For this purpose I refer the reader to comments made in the early chapters regarding differences that can exist in every color viewed neutrally, so that what is called its true color is not changed. For example, gloss can be given to colors or taken away from them without, as a result, blue turning redder or yellower; in other words without changing their kind. Earth colors usually have more strength or denseness but less gloss and vividness than the juice colors. Carmine and Berlin blue already have a degree of both gloss and vividness. These properties can be increased with addition of Calau's wax and, with an addition of binder, an even higher level of gloss can be obtained. Mr. **Calau** has also tried to produce triangles only with earth colors. But I noticed the changes caused by the higher density and the lower gloss and vividness when compared to the colors of the pyramid. I did not find the fact important that earth colors have better coverage and are better suited for painting different kinds of stones and of earths than our colors mixed from carmine, Berlin blue, and gamboge. The latter ones are more like **butterfly colors** because they all have vividness and strength and, together with binder can be made as glossy as one requires. With an addition of lead white or Cremnitz white they approach lighter earth colors more closely while earth colors cannot be made to look like juice colors.

§ 89

If we disregard the difference caused by different degrees of gloss, strength, and vividness in our color pyramid or in paint boxes, we have the complete wealth of colors beginning at the bottom and, in sequential order, distributed into 45, 28, 15, 10, 6, 3, 1 classes and arranged so that each color has the most similar other ones as its neighbors. Between any color and any other one can find the true intermediate grades so that it is possible to introduce more colors between them if it is desired to represent a system with even smaller differences.

§ 90

Earlier, as well as in my *Essay sur la partie photometrique de l'art du peintre*, I have noted that in reduced light blue and red are closer together and approach black more than yellow does. The color pyramid demonstrates this also. The number of steps between the three basic colors is in all the triangles the same, for example 8 in the lowest triangle. But it can be noticed that these 8 steps are less distinct from red to blue than from red, or also blue, toward yellow. The result is that between yellow-green and yellow-red the middle steps appear larger than from yellow-red toward blue-red so that in the former case more intermediate steps can be generated than in the latter. But it was not necessary to introduce such intermediate steps because they can without difficulty be generated. Those interested in this matter can construct the corresponding triangle by intermixing the mixture that belongs to one square with that of the adjacent squares, generating all possible intermediate small steps. In our case it was better to show the main grades still distinctly different from each other.

§ 91

Next, we will view the triangles in the pyramid from the top toward the bottom, because in this manner the origin of the colors becomes more apparent. Accordingly, we have left the only square in triangle 7 white, representing **whiteness** or **light**, thereby reflecting all colored rays simultaneously.

§ 92

In the next lower triangle we have **light** dissolved and separated into its three basic colors **red, blue, yellow,** from the mixture of which all other **prismatic** and **painter's colors** are generated. The only difference is that the latter, because of the admixed shadow, tend more toward **black**, but the prismatic ones, because they are always just **light**, more toward white, as in this case there are no rays interfering with others being reflected from a white object.

§ 93

In the 5th triangle that is next in line we have, aside from the three basic colors, also the intermediate mixtures of always two of them and those are, to the degree possible given the imperfection of the three basic colors used, still prismatic. According to the circular division discussed in chapter 3 these intermediate mixtures fall on Newton's lines separating red and orange, the center of green, and the line separating indigo and violet. The reader will have noted in chapters 6 and 7 that this is the reason I selected them as intermediate mixture: to define thereby the strength of each basic color as well as all other intermediate mixtures.

§ 94

The 4th triangle, just below the 5th, contains in its center, in addition to the prismatic colors on the border, colors always mixed in pairs of two, another color mixed from equal parts of all three basic colors. Applied weakly, it results in a somewhat reddish-brown gray color and is the first example of darkness in painter's colors. As will become more apparent below blue, as a weak prismatic and a painter's color rich in shadow, contributes most to the darkness.

§ 95

In the 3d triangle this tendency toward shade colors divides into three main kinds. On the outline there are 12 colors that are prismatic to the degree that they all consist of mixtures of two basic colors and they resemble the corresponding 12 prismatic colors to the degree allowed by the properties of the three basic colors and the presence of shade in the mixtures. These outer colors surround the three located in the middle, representing the same number of major classes of darker painter's colors. Of these No. 20, bordering on blue, appears black when applied densely, but when applied more diluted it appears gray, but with a brownish cast, even though barely noticeable. Numbers 22 and 33 are brown colors, the first one yellowish, the second reddish and resulting in a slightly reddish chestnut brown. The same numbers but applied more densely are found in the lowest triangle.

§ 96

In the 2nd triangle the earlier mentioned three major classes of darker colors are expanded to 10 classes. Number 51, when applied densely, results in a true black that when applied more diluted is a true gray. Number 58 is the yellowish-reddish gray already noted in 4th triangle. Copper and olive colors begin to be encountered here. At the edge there are 18 colors, mixed only two-by-two from the basic colors. Incidentally, between the third and the second triangle, as well as between the second and the first, there would have been another one if space would have allowed it or if it would truly have been necessary to also show the corresponding colors.

§ 97

As a result, there is only the first triangle left in which the colors have been applied densest, thereby appearing in most cases very dark, except where they border on yellow. Yellow cannot be dark without turning into brown. It is closest to white or to light and at night, in starlight, still very visible and recognizable, while red, brown, blue and black can no longer be distinguished. By the way, even though as dark as most colors in the lowest triangle are in daylight, most can still be distinguished. Only numbers 11, 12, 19, 20 appear fully black and only when applied very weakly show a small difference. Number 11 tends toward blue, No. 19 a bit more toward red, No. 12 slightly toward green, and No. 20 very slightly toward brown. But they are only very minor modifications of gray and they can be used very well as inks. However, No. 51 in the second triangle is even more suitable because, when diluted, it appears pure gray. But here I must remind the reader that these comments only apply in case of the basic colors used for the coloration of the pyramid. If lighter or darker carmine or Berlin blue are used these mixtures will likely appear somewhat different. Poor quality carmine would result in a muddy or dirty black and mess up all the other mixtures.

§ 98

The earlier mentioned black colors are

| | |
|---|---|
| No. 11 | $b^6g^1r^2$ |
| No. 12 | $b^5g^2r^2$ |
| No. 19 | $b^5g^1r^2$ |
| No. 20 | $b^4g^2r^2$ |
| No. 51 | $b^4g^1r^2$ |

From this it is evident that in these mixtures blue is the largest component and thereby is essential in pushing the darkness of the colors all the way to black. This is to be understood to be relative to the strength of each basic color. By weight the following amounts are required for No. 51:

12 parts Berlin blue
12 parts gamboge
2 parts carmine,

thereby equal amounts of blue and yellow but 6 times less red.

§ 99

But how this blackness is generated from the mixture of red, yellow, and blue is a question that deserves to be investigated, all the more because in case of prismatic colors red, yellow, and blue generate not black but white. In this investigation I will proceed **analytically** in the narrowest sense and thereby not derive the experience from sources but the sources from experience. The experience is that a true black is generated from mixture of red, blue, and yellow. It is an old truth in optics that black absorbs all light rays and does not reflect any. Therefore, in the mentioned mixture neither the red nor the blue nor the yellow color particles reflect any light. If they would the mixture would not appear black but tend toward red, yellow, or blue, or in totality toward white. But it is black. From this it follows that the red color particles are prevented by the yellow and blue, the yellow by the blue and red, and the blue by the yellow and red particles to reflect their colored light. Because if this three-fold inhibition would not take place the mixture would reflect red, yellow, and blue rays either in equal or unequal ratio und would thereby appear either white or colored, for example blue-red, green, yellow-red, or blue, yellow, red, etc. But none of this happens. As a result, the earlier mentioned three-fold inhibition must apply. Another conclusion is that the mixture, regardless of how black it appears, must have a few degrees of transparency. This can be concluded from the fact that the mixture, when applied thinly, appears light gray, thereby letting the paper show through and from **Newton**'s experiments showing that thin foils of gold and other materials are transparent. More on this can be found in *Photometria* #617 ff. But let's assume that in the heavily applied, and thereby black-colored, mixture only the particles located on the surface reflect light. There are red as well as blue and yellow particles on the surface. Because they are not covered by anything each of these reflects its own specific light and as a result again white or another color appears, but not black. Therefore, it cannot be valid that light is only reflected from the surface. The heavily applied mixture must be transparent to some extent so that light penetrates it and to the degree the light is reflected is emitted. But I have shown already that nothing is emitted. And this can be explained as follows. Assume a particle of red below the surface so that it is illuminated by light. Of the total light the particle only reflects the red rays. But before these red rays reach the surface they are captured and absorbed by the blue and yellow color particles because they do not reflect red light rays. In the same manner the yellow light reflected from yellow particles is lost in the red and blue particles and the blue light reflected from blue particles in the yellow and red particles. What is reflected all the way to the surface is a very small and scattered portion of colored light of which there are a few weak traces visible in form of little spots of rainbow colors when exposing a black polished object to sun light. This weak and scattered light demonstrates that there is a difference in blackness between black objects and darkness. The latter has no light, the former reflects a tiny amount.

§ 100

Transparent colored objects, such as glass, have already been subjected to similar examination. If a sheet of glass that only transmits red light is placed on top of one letting pass only blue or yellow rays results in only little or even no light passing simultaneously through both of them. Blue glass only lets blue rays pass through, yellow glass only yellow rays. Both kinds did not pass through red glass and, as a result, little or no light passes through. The result is the same if in place of the red glass the yellow or the blue one is used. To be most successful all three kinds of glass should be used because, in combination, the glasses are not even minimally transparent for red, yellow, and blue light.

§ 101

It should be mentioned in regard to black mixtures that all three basic colors are necessary to achieve them because mixtures of two of them result, as in the prism, only always in an intermediate color. Such colors can be a very dark blue-green or blue-red if more blue than yellow or red are used in the mixture. But that does not make them black. So that the colored rays of all three basic colors are completely absorbed both of the other two basic colors are required in the appropriate ratio. From this one might conclude that some of the red rays are more easily absorbed by yellow, others more easily by blue, and so on. However, it can be conjectured that the three kinds of color particles in the mixture arrange themselves, as a result of their form, in such a manner that the colored rays of each particle loose themselves more easily and completely in the particles of the two other colors.

## Chapter 11 Naming of colors

§ 102

To make progress in the rather difficult matter of naming of colors, even those with the smallest differences, Schäfer and Schiffermüller offered several ideas. Myself, I will make an effort to bring basic reasoning into this subject. That the concept of colors cannot be explained to a blind person with simple words has been mentioned several times as an example in the theory of logic. Names are only for the purpose of recalling an earlier learned experience to memory. Accordingly, colors have to be experienced first, because otherwise their names are of little help.

§ 103

This is true in the narrowest sense for the main kinds of colors, **white, red, yellow, green, blue, brown, black**. And this brings up the question of their mixtures, as the nature of their appearance is to become clear from their names. Each mixture borrows something from the mentioned seven main kinds. This needs to be indicated so as to provide an accurate designation. We begin with the lowest triangle and in that with the series from yellow to blue. Among all colors these are the ones for which it is easiest to find an appropriate name. Here we have

No. 1 Blue
No. 2 Greenish blue
No. 3 Green-blue or blue-green
No. 4 Bluish green
No. 5 Green
No. 6 Yellowish green
No. 7 Yellow-green or green-yellow
No. 8 Greenish yellow

No. 9 Yellow

It is clear that these names are meaningful, because in the lowest triangle the colors are only mixed in 1/8 parts and their number is thereby limited. If between each two of them one would want to name an intermediate color, it would be almost necessary to do what sailors do in naming their 32 or 64 kinds of wind. Naming is simplified by the fact that **green** falls in the middle between **blue** and **yellow**, a color showing up as a main kind in our list, treated in languages in the same manner as are red, blue, and yellow.

§ 104

Language is less helpful for the series ranging from **red** to **blue**. The central color No. 31 is red-blue or blue-red, neither red enough to be named **violet** nor blue enough to be **purple**. If, therefore, we only wish to use red and blue for the names we would have to proceed as sailors do and name

No. 1 Blue
No. 10 Blue toward red
No. 18 Blue-red-blue
No. 25 Blue-red toward blue
No. 31 Blue-red or red-blue
No. 36 Red-blue toward red
No. 40 Red-blue-red
No. 43 Red toward blue
No. 45 Red

I do not see why one could not get used to these terms as easily as sailors have gotten used to the names of the winds. Similar names would result for the series from red to yellow:

No. 9 Yellow
No. 17 Yellow toward red
No. 24 Yellow-yellow-red
No. 30 Yellow-red toward yellow
No. 35 Yellow-red or red-yellow
No. 39 Red-yellow toward red
No. 42 Red-red-yellow
No. 44 Red toward yellow
No. 45 Red

However, the wind directions are easier to distinguish, as they are located on a circle, than the color steps, especially if they are not placed as a series before the viewer. Without that considerable training is required.

§ 105

With all that we have only covered the colors located on the outer border of the triangle. But there are another 21 colors within the triangle for which scientific names also need to be found, especially because in this region only the names **brown** and **black** are common and in language are treated comparable to **red, blue, yellow, green, white** or **light**. But black and brown are insufficient in regard to naming and we, therefore, have to employ additional color names that are reasonably well known. Accordingly, we have in sequence

No. 11 Bluish black
No. 12 Greenish black
No. 13 Dark-shaded black-green
No. 14 Shaded blackish green
No. 15 Brown-green or olive
No. 16 Wilting leaves before they turn yellow-red
No. 17 Reddish yellow

When green nutshells dry they pass pretty much through steps 16, 5, 14, 13, 12 before they turn black.

No. 19 Reddish black
No. 20 Brownish black
No. 21 Black-brown
No. 22 Blackish brown, umbra
No. 23 Yellowish brown

And further

No. 26 Blackish purple
No. 27 Chestnut red-brown
No. 28 Strong brown
No. 29 Copper red

No clearly indicative names can be found for the following numbers. But we can arrange the series differently and then we find in the series No. 1 to No. 45 and in No.s 41, 37, 32, 26, 19, 11 all kinds of blue-red, blue, black-blue, etc., plum colors, as in part can still be seen in No. 27, 33, and 38. Cherry, apple, and pear colors are found in the front rows. New, old, and rusty copper, all colors of new and old roof tiles, moss colors, shadowy tree leaf colors, etc., can be found without difficulty. In No. 42 and 39 we have cinnabar colors and in the same series lead red, orpiment, orange, golden yellow, lemon yellow, as well as in the series 1 to 45 all kinds of purple, indigo, and violet colors. The lighter of these colors are found in the upper triangles, among these especially also the rose colors. Colors are applied at such a density in the lowest triangle that they must be observed in bright daylight to notice their differences more clearly.

## Chapter 12 Comparison of color mixtures by calculation

§ 106

The colors of the triangles in the color pyramid are arranged, as is apparent, in three kinds of series. These series are parallel to the sides of the triangle. It is easily seen that in each of these series a color transitions toward the other one, eventually disappearing in it. If one wants to find the color intermediate between two colors that are not located in such a series a straight line can be drawn between them and this line will pass through one, two, or more squares. The colors of these squares are the closer to the searched-for intermediate grade the closer the line falls to the center of a square. But in most cases they are already somewhat different from the desired middle point.

§ 107

But if the problem is to be solved more accurately and if it is desired to have between any two mixtures a particular number of intermediate steps the two reference mixtures can be taken as basic colors, to be treated as we have treated the three true basic colors. For example, let's designate No. 6 and No 26 as the two reference mixtures and we want to have 6 intermediate grades between them. Designating the first A, the second B we have

$$A^7, A^6B^1, A^5B^2, A^4B^3, A^3B^4, A^2B^5, A^1B^6, B^7$$

and thereby six intermediate grades for the two defined mixtures. As shown in paragraph 80

$$A = b^1g^6r^1$$
$$B = b^4g^1r^3$$

This results in the following grades

$$A^7 = b^7g^{42}r^7$$
$$A^6B^1 = b^{10}g^{37}r^9$$
$$A^5B^2 = b^{13}g^{32}r^{11}$$
$$A^4B^3 = b^{16}g^{27}r^{13}$$
$$A^3B^4 = b^{19}g^{22}r^{15}$$
$$A^2B^5 = b^{22}g^{17}r^{17}$$
$$A^1B^6 = b^{25}g^{12}r^{19}$$
$$B^7 = b^{28}g^7r^{21}$$

$A^4B^3$ indicates that four times the amount of A which is $b^4g^{24}r^4$ needs to be added to three times the amount of B which is $b^{12}g^3r^9$ with the resulting total being $b^{16}g^{27}r^{13}$. I have mentioned already earlier that this type of expression has been selected by **Mayer** only because of its simplicity. To use proper algebraic expression one would need to write

$$A = b + 6g + r$$
$$B = 4b + g + 3r \text{ and thereby}$$
$$4A + 3B = 16b + 27g + 13r.$$

All the other intermediate grades would have to be expressed in the same fashion. The coefficients, or according to **Mayer** the *partientes,* only represent the portions and first they have to be converted to weights after, as discussed in chapter 7, one has determined how the basic colors r, g, b relate to each other in regard to strength.

§ 108

By this method it is possible to determine between two colors of choice a mixture that is in regard to the ratio closer to one than to the other. For example, color #41 of the lowest triangle is to be mixed with black #51 of the second triangle so that the mixture contains 2/5 black and 3/5 of color #41. Here we have

$L = b^1g^1r^6$

$S = b^4g^1r^1$ and as a result the desired mixture is

$S^2L^3 = b^8g^2r^2 + b^3g^3r^{18} = b^{11}g^5r^{20}$

§ 109

To determine which color in the lowest triangle comes closest to this color the sum of the parts is calculated: 11 + 5 + 20 = 36. It is evident from this that the mixture is calculated in 1/36 parts. But on the lowest triangle the colors are calculated in 1/8 parts and as a result we need to proportionately reduce the one ratio to the other.

36 : 8 = 11 : 2 4/9

36 : 8 = 5 : 1 1/9

36 : 8 = 20 : 4 4/9.

Without a noticeable error we can round these numbers to 2.5, 1, and 4.5. The mixture calculated in 1/8 parts is therefore $b^{2.5}g^1r^{4.5}$ and it falls between $b^3g^1r^4$ and $b^2g^1r^5$ and thereby halfway between #32 and #37. If black #11 would have been used instead of black #51 L and S would be

$L = b^1g^1r^6$

$S = b^6g^1r^1$ and thereby

$S^2L^3 = b^{12}g^2r^2 + b^3g^3r^{18} = b^{15}g^5r^{20}$.

This mixture, reduced to 1/8 parts results exactly in $b^3g^1r^4$ and thereby in color #32.

§ 110

In exactly the same manner it is possible to calculate mixtures from three and more colors which themselves are results of mixtures. For example, let's mix #1 for blue, #17 for King's yellow, and #39 for cinnabar to see what a triangle mixed from these in 1/6 parts would look like. It can easily be seen that many colors would not be present in it. When drawing straight lines through the center of squares 1, 17, 39 they form a triangle and the mixtures located outside this triangle cannot be produced with colors # 1, 17, 39. The calculation is as follows

# 1 = $B = b^3$

# 17 = $G = g^7r^1$

# 39 = R = $g^3r^5$ .

Because the triangle is to be calculated in 1/6 parts the following is obtained

| | | | | | | |
|---|---|---|---|---|---|---|
| $B^6$ | | | | | | |
| $B^5G^1$ | $B^5R^1$ | | | | | |
| $B^4G^2$ | $B^4G^1R^1$ | $B^4R^2$ | | | | |
| $B^3G^3$ | $B^3G^2R^1$ | $B^3G^1R^2$ | $B^3R^3$ | | | |
| $B^2G^4$ | $B^2G^3R^1$ | $B^2G^2R^2$ | $B^2G^1R^3$ | $B^2R^4$ | | |
| $B^1G^5$ | $B^1G^4R^1$ | $B^1G^3R^2$ | $B^1G^2R^3$ | $B^1G^1R^4$ | $B^1R^5$ | |
| $G^6$ | $G^5R^1$ | $G^4R^2$ | $G^3R^3$ | $G^2R^4$ | $G^1R^5$ | $R^6$ |

By assigning the following values for B, G, R

$B = b^8$

$G = g^7r^1$

$R = g^3r^5$

and reducing them the desired triangle is obtained

| | | | | | | |
|---|---|---|---|---|---|---|
| $b^{48}$ | | | | | | |
| $b^{40}g^7r^1$ | $b^{40}g^4r^4$ | | | | | |
| $b^{32}g^{14}r^2$ | $b^{32}g^{10}r^6$ | $b^{32}g^6r^{10}$ | | | | |
| $b^{24}g^{21}r^3$ | $b^{24}g^{17}r^7$ | $b^{24}g^{13}r^{11}$ | $b^{24}g^9r^{15}$ | | | |
| $b^{16}g^{28}r^4$ | $b^{16}g^{24}r^8$ | $b^{16}g^{20}r^{12}$ | $b^{16}g^{16}r^{16}$ | $b^{15}g^{12}r^{20}$ | | |
| $b^8g^{35}r^5$ | $b^8g^{31}r^9$ | $b^8g^{27}r^{13}$ | $b^8g^{23}r^{17}$ | $b^8g^{19}r^{21}$ | $b^8g^{15}r^{25}$ | |
| $g^{42}r^6$ | $g^{38}r^{10}$ | $g^{34}r^{14}$ | $g^{30}r^{18}$ | $g^{26}r^{22}$ | $g^{22}r^{26}$ | $g^{18}r^{30}$ |

Here the mixtures are in 1/48 parts and therefore need to be divided by 6 if one wants to compare them with the colors of the lowest triangle in the pyramid, or by 8 if they are to be compared with those of the second triangle in the pyramid. For example the mixtures $b^{32}g^{10}r^6$, $b^{32}g^6r^{10}$ are very close to $b^5g^2r^1$, $b^5g^2r^1$ and thereby close to # 12 and # 19 of the lowest triangle, as well as to $b^4g^1r^1$ or # 51 of the second triangle. The difference becomes apparent when calculating the mixtures of this number in 1/48 parts. Then

# 12 = $b^{30}g^{12}r^6$, therefore $b^{32}g^{10}r^6$ hastoo much blue by 1/24 and not enough yellow by 1/24;

# 19 = $b^{30}g^6r^{12}$, therefore $b^{32}g^6r^{10}$ has 1/24 too much blue and not enough red by1/24;

# 51 = $b^{32}g^8r^8$, therefore $b^{32}g^{10}r^6$ has too much yellow by 1/24 and not enough red, but $b^{32}g^6r^{10}$ has not enough yellow by 1/24 and too much red.

As a result # 51 is half way between $b^{32}g^{10}r^{6}$ and $b^{32}g^{6}r^{10}$. If the two colors are mixed in equal parts $b^{64}g^{16}r^{16}$ is obtained which is identical to $b^{4}g^{1}r^{1}$. By the way, if I say here that the two colors are different by 1/24, this is related to the total mixture, calculated in 1/48 parts. The reason is that in $b^{32}g^{10}r^{6}$ yellow g has a value of $g^{10}$ instead of $g^{8}$ and red $r^{6}$ instead of $r^{8}$. In each case the difference is 2, therefore 2/48 or 1/24 of the total mixture. The two differences add up to 4/48 or 1/12, and by that value, as a sum, the mixtures $b^{32}g^{10}r^{6}$, $b^{32}g^{6}r^{10}$ deviate from # 12, 19, 51 of the two lower triangles. In these blackish colors the difference is barely noticeable and this makes it clear that, in his triangle, **Mayer** was able to obtain ivory black. His cinnabar, Royal yellow, and azurite are in very good agreement with #17, 39, 1 except that azurite, because of its lightness already contains much white and thereby can be compared only with very lightly painted Berlin blue. But **Mayer** thinks that Berlin blue contains 1/12 of red because he gave it the composition $r^{1}b^{11}$. This may be due to the fact that Berlin blue is very dark and pointing in the direction of black. But in consequence there would also need to be some yellow present in it, because blue and red alone do not produce black. I do not consider

Berlin blue to be the perfect, pure blue, however in the two kinds I tested I did not notice any redness. I have not seen the **Mayer's** Berlin blue cannot make a judgment about it. By the way, 1/12 of red in blue is hardly noticeable, especially if there is no whiteness or lightness present in it. Then it is quite specifically and strongly blue. But that cinnabar, when compared to true red, contains much yellow is easily noticeable in the lowest triangle. As a result, blood and carmine colors, in addition to the more beautiful violet and purple colors, are missing in Mayer's triangle.

§ 111

Because carmine is an expensive color, in its stead frequently Florentine lake is used because it is much less expensive. In the manner just described we can determine to what degree the latter can replace the former. Florentine lake varies in quality, in some cases it is more intensive, in others it is more bluish. For the purpose of this example we will assume color # 43 to be representative. Selecting in place of yellow # 9 greenish yellow # 8 as basic yellow color and Berlin blue as basic blue and mixing them in 1/7 parts, all colors located in the triangle # 1, 8, 43 are obtained so that only the lowest row, # 9 to # 45 is lost. A general proof for this is possible. We have

$$\# 1 = B = b^{8by}$$
$$\# 8 = G = b^{1}g^{7}$$
$$\#43 = R = b^{1}r^{7}.$$

Each mixture can be represented in 1/7 parts by

$$B^{m}G^{n}R^{7-m-n}$$

where m and n are whole numbers that together cannot exceed the value 7.

But

$$B^{m} = b^{8m}$$
$$G^{n} = b^{n}g^{7n}$$
$$R^{7-m-n} = b^{7-m-n}r^{49-7m-7n}.$$

As a result the complete mixture is

$$B^mG^nR^{7-m-n} = b^{8m+n-m-n}g^{7n}r^{49-7m-7n}$$
$$= b^{7m+7}g^{7n}r^{49-7m-7n}.$$

This mixture is identical with $b^{m+1}g^nr^{49-7m-7n}$ because each portion can be reduced to 1/7. If m + n does not exceed the value of 7, the mixture $b^{m+1}g^nr^{7-m-n}$ represents every color of the lowest triangle that is not located in the front row # 9 - - # 45. As a result it is immediately apparent which mixtures of colors # 1, 8, 43, mixed in 1/7 parts, are generated.

§ 112

When using gamboge # 9 instead of combining color #1 and # 43 with the greenish yellow # 8, one will not obtain any of the colors # 17, 24, 30, 35, 39, 42, 44, 45 but only the early numbers in approximation. The reason is that a straight line passing through the center of the squares of # 9 and # 43 does not pass through the square of # 45 at all und through the others only above their centers. It would therefore be necessary to calculate and paint one or several triangles based on

$$B = b$$
$$G = g$$
$$R = b^1r^7$$

as it has been described earlier for the three basic colors, or just above (§ 110) for colors # 1, 17, 39.

§ 113

The darkness of the colors painted in the lowest triangle and containing all three basic colors can be viewed as the result of the included blackness, as if they would consist of black and a prismatic color located on the outer border of the triangle. The amount of black can also be calculated. I mentioned already earlier that black # 51 applied densely is the strongest black. It consists of $b^4r^1g^1$. Let's consider any desired mixture of the lowest triangle consisting of all three basic colors

$$b^mg^nr^{8-m-n}$$

where m and n are whole numbers with a sum not larger than 7. In order to avoid fractions we want to multiply these portions with 4, obtaining

$$b^{4m}g^{4n}r^{32-m-n}.$$

These mixtures are calculated in 1/32 parts while black $b^4g^1r^1$ has 6 parts. Now, the 6-portion black $b^4g^1r^1$ is subtracted from $b^{4m}g^{4n}r^{32-4m-4n}$ as many times until 0 is obtained for either blue, yellow, or red, a situation that in case of two colors can happen simultaneously. As many times as the amount of 6-portion black was subtracted they are added to the remaining portions of colors and the result is how any mixture can be taken as the result of black and a prismatic color. The example of $b^2g^3r^3$ is multiplied times 4, resulting in

$$b^8g^{12}r^{12}.$$

From that formula black $b^4g^1r^1$ can be subtracted two times, with $g^{10}r^{10}$ remaining. To this 2 times 6-portion black is added, with the result

$$b^8g^{12}r^{12} = s^{12}g^{10}r^{10}.$$

Color $b^4g^1r^3$ multiplied times 4 is $b^{16}g^4r^{12}$ from which black $b^4g^1r^1$ can be subtracted 4 times, with $r^8$ remaining. To this 4 x 6 = 24 portions are added because of the 4 times subtracted black. As a result we have

$$b^{16}g^4r^{12} = s^{24}r^8 .$$

Here I am showing all 21 mixtures of the lowest triangle that contain all three basic colors

| | | | | | |
|---|---|---|---|---|---|
| $b^{24}r^4g^4$<br>$s^{24}b^8$ | | | | | |
| $b^{20}g^8r^4$<br>$s^{24}g^4b^4$ | $b^{20}g^4r^8$<br>$s^{24}b^4r^4$ | | | | |
| $b^{16}g^{12}r^4$<br>$s^{24}g^8$ | $b^{16}g^8r^8$<br>$s^{24}g^4r^4$ | $b^{16}g^4r^{12}$<br>$s^{24}r^8$ | | | |
| $b^{12}g^{16}r^4$<br>$s^{18}g^{13}r^1$ | $b^{12}g^{12}r^8$<br>$s^{18}g^9r^5$ | $b^{12}g^8r^{12}$<br>$s^{18}g^5r^9$ | $b^{12}g^4r^{16}$<br>$s^{18}g^1r^{13}$ | | |
| $b^8g^{20}r^4$<br>$s^{12}g^{18}r^2$ | $b^8g^{16}r^8$<br>$s^{12}g^{14}r^6$ | $b^8g^{12}r^{12}$<br>$s^{12}g^{10}r^{10}$ | $b^8g^8r^{16}$<br>$s^{12}g^6r^{14}$ | $b^8g^4r^{20}$<br>$s^{12}g^2r^{18}$ | |
| $b^4g^{24}r^4$<br>$s^6g^{23}r^3$ | $b^4g^{20}r^8$<br>$s^6g^{19}r^7$ | $b^4g^{16}r^{12}$<br>$s^6g^{15}r^{11}$ | $b^4g^{12}r^{16}$<br>$s^6g^{11}r^{15}$ | $b^4g^8r^{20}$<br>$s^6g^7r^{19}$ | $b^4g^4r^{24}$<br>$s^6g^3r^{23}$ |

Each square contains on top the formula for the mixtures of the three basic colors. Below them is the formula for the same colors mixed from black and one or two of the basic colors, demonstrating how black produces their darkness.

## § 114

Among these mixtures are three that in addition to black contain only one basic color:

$s^{24}b^8$ which is # 11
$s^{24}g^8$ which is # 13
$s^{24}r^8$ which is # 26.

In all three of them black is three times as strong as the basic color. Compared to that the mixtures $s^{12}g^{14}r^6$ which is # 22
$s^{12}g^6r^{14}$ which is # 33 are closest to the balanced mixtures $s^1g^1$ and $s^1r^1$ so that # 22 is closest to true yellow-black or brown and # 33 closest to black-red or chestnut brown. More accurately, the following mixtures in 1/12 part apply

$$s^6g^6 = b^4g^1r^1 + g^6 = b^4g^7r^1$$
$$s^1r^1 = b^4g^1r^1 + r^6 = b^4g^1r^7.$$

## § 115

A given mixture of red, yellow, and blue cannot always be reproduced with three other mixtures that also contain red, yellow, and blue. Let's take # 28 as the mixture to be matched with # 5, 15, 24. The following applies

$$\# 28 = b^2g^3r^3$$
$$\# \ \ 5 = b^4g^1$$
$$\# 15 = b^2g^5r^1$$
$$\# 24 = g^6r^2 .$$

Taking # 5 x times, # 15 y times and # 24 z times the resulting mixture is

$$b^{4x}g^{4x} + b^{2y}g^{5y}r^y + g^{6z}r^{2z} .$$

The mixture consists of $b^{4x+}2y + g^{4x+5y+5z} + r^{y+2z}$ and it is supposed to be $b^2g^3r^3$. From this we have

$$4x + 2y = 2$$
$$4x + 5y + 6z = 3$$
$$y + 2z = 3.$$

If the first of these equations is subtracted from the second the result is

$$3y + 6z = 1;$$

but the third equation states $3y + 6z = 9$ and so 1 should equal 9 which does not make sense. The reason for this is easy to see: the three colors # 5, 15, 24 contain too much yellow and as a result the significant amount of red in # 28 cannot be matched without adding more yellow than what is required for # 28.

§ 116

We will turn the problem around and determine with what other mixtures of the lowest triangle it would be possible to match mixture # 28. The latter consists of $b^2g^3r^3$. There are different possible solutions to this problem.

1. If it is to be matched with two mixtures. In this situation we double the portions to result in $b^4g^6r^6$. The problem is limited to how the numbers 4 + 6 + 6 can be separated in some manner into two equal parts. This is possible in 15 different ways as follows.

| | | |
|---|---|---|
| $b^4g^4 + g^2r^6$ | $b^3g^5 + b^1g^1r^1$ | $b^2g^6 + b^2r^6$ |
| $b^4g^3r^1 + g^3r^5$ | $b^3r^4g^1 + b^1g^2r^5$ | $b^2g^5r^1 + b^2g^1r^5$ |
| $b^4g^2r^2 + g^4r^4$ | $b^3g^3r^2 + b^1g^3r^4$ | $b^2g^4r^2 + b^2g^2r^4$ |
| $b^4g^1r^3 + g^5r^3$ | $b^3g^2r^3 + b^1g^4r^3$ | |
| $b^4r^4 + g^6r^2$ | $b^3g^1r^4 + b^1g^5r^2$ | |
| | $b^3r^5 + b^1g^6r^1$ | |

Thus, the mixture $b^4g^6r^6$ can be separated in 15 different ways into two mixtures from the lowest triangle and can also be composed by mixing these in equal parts.

2. But if the two mixtures are not taken in equal parts there are additional solutions. Let's state that the ratio between the two is to be 2 to 3. To solve this, 2 + 3 = 5 and one multiplies each portion by 5 and takes them five-fold, that is $b^{10}g^{15}r^{15}$. Now 10 + 15 + 15 is separated as many ways as possible so that one part has 16 portions and the other 24

and the portions of basic colors can be divided by 2 in the former case and by 3 in the latter. In this manner the following is obtained

$b^{10}g^6 + g^9r^{15}$, which is equal to 2 times $b^5g^3$ + 3 times $g^3r^5$,

$b^4g^6 + b^3g^3r^{15}$, which is equal to 2 times $b^2g^3$ + 3 times $b^1g^1r^5$.

III. If color $b^2g^3r^3$ is to be composed of three other mixtures in equal parts we use the components threefold, that is, $b^6g^9r^9$ and divide 6 + 9 + 9 in all possible ways into three parts, e.g.,

$$b^6g^2 + g^7r^1 + r^8$$
$$b^6g^1r^1 + g^5r^3 + g^2r^5 \text{ etc.}$$

It is easy to see that also here one can make the portions uneven and make mixtures of 4, 5 and more others, either in equal or unequal parts to match mixture $b^2g^3r^3$ or any other one. But it is apparent that these solutions are just make-work because several solutions with two mixtures are available and it is not necessary to find special cases when mixing various already mixed colors because fundamentally the mixtures are always made from red, blue, and yellow, independent of how the colors used for the mixtures may be different in appearance to those basic colors (this excludes water that in one way represents light, in another it is a fourth basic color).

## § 117

Here is one more example involving black # 51 = $b^4g^1r^1$and how matches based on colors of the lowest triangle can be calculated. It is not possible to do this with two colors in equal parts. To do so requires the mixtures

$$b^3 + b^1g^1r^1$$
$$b^2g^1 + b^2r^1.$$

But these are calculated in 1/3 parts while colors of the lowest triangle are calculated in 1/8 parts. But if three colors are used or one is used twice there are several possible solutions, such as

$b^2 + b2 + g^1r^1$ produces in the lowest triangle 2 times $b^8g^4r^4$,

$b^1g^1 + b^2 + b^1r^1$ produces in the lowest triangle $b^4g^4 + b^8 + b^4r^4$.

By quadrupling these portions, that is $b^{16}g^4r^4$ , the mixture can be divided into three equal parts in several different ways:

| | | |
|---|---|---|
| $b^8 + b^8 + g^4r^4$ | $b^8 + b^5g^3 + b^3g^1r^4$ | $b^7g^1 + b^7g^1 + b^2g^2r^4$ |
| $b^8 + b^7g^1 + b^1g^3r^4$ | $b^8 + b^5g^2r^1 + b^3g^2r^3$ | $b^7g^1 + b^7r^1 + b^2g^3r^3$ |
| $b^8 + b^7r^1 + b^1g^4r^3$ | $b^8 + b^5gr^2 + b^3g^3r^2$ | $b^7r^1 + b^7r^1 + b^2g^4r^2$ |
| $b^8 + b^6g^1r^1 + b^2g^3r^3$ | $b^8 + b^5r^3 + b^3g^4r^1$ | $b^7g^1 + b^6g^2 + b^3g^1r^4$ |
| $b^8 + b^6g^2 + b^2g^2r^4$ | $b^8 + b^4g^4 + b^4r^4$ | $b^7g^1 + b^6g^1r^1 + b^3g^2r^3$ |
| $b^8 + b^6r^2 + b^2g^4r^2$ | $b^8 + b^4g^3r^1 + b^4g^1r^2$ | $b^7g^1 + b^6r^2 + b^3g^3r^2$ |
| | $b^8 + b^4g^2r^2 + b^4g^2r^2$ | $b^7r^1 + b^6g^2 + b^3g^2r^3$ |
| | | &c. |

§ 118

If such lists are to be complete and represented by generalized formulas, we find several problems that are really **Diophantine** in nature. In this connection especially also the possibility of matching a given mixture with 2, 3, etc. other mixtures must be investigated. But I will not concern myself here with this problem. Instead I will show in the form of an example the already mentioned impossibility of matching color $b^4g^1r^1$ with two colors from the lowest triangle taken in equal parts. For this purpose I am taking color $b^4g^1r^1$ *x* times, that is $b^{4x}g^xr^x$ . The two colors from the lowest triangle that we are looking for are to be taken *y* times

$$b^{ny}g^{my}r^{(8-m-n)y}$$
$$b^{py}g^{qy}r^{(8-p-q)}.$$

The values of x, y, m, n, p, q are to be whole numbers and n + m < 8 and also p + q < 8. The two colors together are to result in color $b^{4x}g^xr^x$. As a result we have three equations

$$ny + py = 4x$$
$$my + qy = x$$
$$16y – my – ny – py – qy = x.$$

From these it follows that

$$n + p = 4x/y$$
$$m + q = x/y$$
$$16 – m – n – p – q = m + q$$

therefore $16 = 2m + 2q + n + p.$

From the first two equations it follows that

$$n + p = 4m + 4q$$

and so we have $n + p = 32 – 2n – 2p$

and as a result $n + p = 32/3.$

But n and p are supposed to be whole numbers. Therefore n + p cannot be a fraction and the desired mixture is not possible.

§ 119

But if the two colors from the lowest triangle are not to be in equal parts but just to relate as one whole number to another whole number we have

$$b^{4x}g^xr^x = b^{my}g^{ny}r^{(8-m-n)y} + b^{pz}g^{qz}r^{(8-p-q)z}$$

and thereby the three equations

$$4x = my + pz$$
$$x = ny + qz$$
$$x = 8y – my – ny + 8z – pz – qz .$$

From these it follows that

$$my + pz = 4ny + 4qz$$
$$0 = 8y + 8z – my – pz – 2ny – 2qz .$$

And from these $16y + 16 z = 3my + 3pz$

or $my + pz = (16y + 16z)/3$ = a whole number.

As a result y + z must be divisible by 3 and also (my + pz) divisible by 16. Because ny + qz = ¼ (my+pz), n and q of these equations have to be determined in agreement with this equation. In addition, m + n, p + q cannot be larger than 8. If, for example, y = 4, z = 5 the following is obtained

$$4m + 5p = 48$$
$$4n + 5p = 12 .$$

The first of these equations can have only two values

1. m = 2, p = 8
2. m = 7, p = 4

because m and n must be whole numbers and they cannot be larger than 8. For the same reasons the second equation can only have one value

$$n = 3, q = 0 .$$

This value cannot be combined with the second one of those found earlier: m = 7, p = 4, because m + n = 3 + 7 = 10, thereby larger than 8. As a result the only possibility remaining is

$$m = 2, n = 3$$
$$p = 8, q = 0 .$$

From this we obtain

$$b^{4x}g^{x}r^{x} = b^{8}g^{12}r^{12},$$

two mixtures consisting of four times $b^2g^3r^3$ and five times $b^8$, thereby consisting of four times # 28 and five times # 1. The result will be 12 times mixture # 51, $b^4g^1r^1$, because x will be 12. However, I will not pursue this calculation further.

## Chapter 13 Use of the color pyramid

§ 120

In the beginning, I will limit myself to considering the color pyramid only as a collection of all colors separated by small steps with the stronger colors separated into 45, or the darker and lighter colors together into 108 compartments or classes. Viewed in this way the collection represents a generalized color sample chart, useful to merchants to determine if they have fabrics, towels, silks, camel hair, threads, etc., of all colors or if in addition to those painted in the pyramid they require intermediate grades to the extent they can be distinguished. I have mentioned already earlier that especially between yellowish colors it is possible to think of additional intermediate grades. Just viewing the samples also shows this.

§ 121

Customers can make even more use of the pyramid as a color chart than merchants. The latter need not have fabrics, silks, wool, yarns, of all colors. But it is the former who decide what colors they want to select or must have for a special purpose. Color assortment or color combination in clothing is much simplified using the color pyramid. It applies to any fashion and offers the right colors for any age, class, or sex. Selinda wants to have a dress like Caroline's. She takes note of the number and triangle in which the color appears and can be sure to get the same color. There is no difficulty should the color be lighter or darker or tend more in direction of one or another color. The color pyramid ranges through all colors from carbon black to snow white. That it will be of considerable service in embroidery of flowers, leaves, fruit, etc. is quite obvious.

§ 122

It is not sufficient to concern ourselves only with merchants and buyers. We also have to consider dyers. For them it does not suffice to just have a general color chart available. They are supposed to be able to dye any kind of fabric according to any of the samples in the system. To achieve this often requires many trials and much thought, especially when the dyed fabrics must be fast to rain, sun, humid and dry air, etc. and should not change in washing. The best advice is that they find a **blue, red**, and **yellow** dye with which the true intermediate colors can be achieved and then determine the weights required to obtain the intermediate mixtures where each dye must have the same **degree of strength**. In short, they must do everything that I have done above in regard to **carmine, Berlin blue**, and **gamboge** related to watercolors in order to be able to develop the color pyramid using their dyes. It therefore will not be necessary to repeat the process here.

§ 123

The color pyramid is not just a color chart but in it colors are arranged according to how they relate and by what steps. This helps the memory to a very considerable degree. Without bothering about the names of colors one can fix in memory their location or place in the color pyramid in such a way that one retains its image in the mind and can for any color encountered recall its location in one triangle or another, even if the triangles are not present.

§ 124

To practice painting flowers, fruits, animals, insects, or even complete landscapes according to nature one begins with drawing itself using, where it is necessary, the rules of linear perspective. There is no need to be concerned about reproducing the colors. They are looked up in the pyramid and their numbers are written next to the objects. It is then possible to return home and be certain not to have forgotten the colors, but that they can be reproduced. Aerial perspective does not pose a problem. A forest or mountain in the distance might appear bluish, green, dark or light blue. There is no problem finding the same bluish green, dark or light blue color in the pyramid. The only thing that needs to be considered is the difference in the light falling on the landscape. If a landscape lit by the pink light of the rising sun is to be painted all colors tend toward red, except those that are not lit by this light but are in the shade. In this situation the pyramid should not itself be lit by the pink morning light because this light would change the colors also. Instead, the pyramid must be held in such a way that light containing redness but also the normal blueness falls on it. If this cannot be achieved, the way to take consideration of the morning light is to add an amount of red to each color so that the influence of the reddish morning light is properly represented. I say: **add**. Because the morning redness makes the colors of the pyramid appear more reddish than they are in normal daylight and they are in fact less reddish than they appear. For this reason it is necessary to add to them one or a few parts of red, depending on how widely the morning redness is distributed in the sky. A similar kind of change may be required in regard to the blueness of the light of the sky that alone falls on the pyramid when it is kept in the shade. The best light is that from a sky covered with light, white clouds, because such light changes colors to the least degree from what they are.

§ 125

When using the color pyramid no difficulties occur in regard to shadowing in painting. Everything depends on the quality of the light that illuminates those parts located in the shadow of the stronger light. If the shadow light is white and the colors of the objects in the shadow are light it is only necessary to move from upper triangles to lower ones; in other words, it suffices to apply the color of the lighted object more lightly, when in the shade more strongly and to add a little amount of black or gray to white. But if the actual color of the object in the shade is strong, so that it is found in the lowest triangle, it is located there and then a straight line is drawn from its center to the center of the four black colors 11, 12, 19, 20. This line will represent the looked-for shadow colors and, depending on the degree of shade the one that represent the actual situation can be selected. But when objects in the shade are illuminated with light of the blue sky or light reflected from greening trees the mentioned line from the true color of the object will not be drawn toward black but toward blue or green or, in general, toward the color of the light illuminating the objects in the shade. As a result the line will pass over the colors of which one is required to paint a particular object. Which one is to be used depends in part on the strength, in another on the brightness of the light and particularly on the color of the object itself. If, for example, the object is white it will have the color of the illuminating light but a few degrees lighter. If it has the same color as the light, the color will not change, only become stronger and more vivid. If the object is black it will remain black and in the shade will appear the blacker the darker the colored light is by itself, in other words, the more it is dark blue, dark green, dark red, etc. It is easy to discern that the closer the color of the object is to white or yellow, the more it will accept color from the illuminating blue or red light. The more the object is reddish or bluish the less influence yellow or light-green light has on its color because the color of the object itself is already dark to a considerable degree. Those who may want to do special tests with colored papers to see how one reflects its color onto another, resulting in mixed colors will find the color pyramid useful as a general yard stick. In such a situation it is necessary to keep in mind variations in brightness because brightness does not change the hue of the color. The colored light illuminating an object or a part of it has its source often nearby. All that is needed is that two areas of different color are close to each other, such as in clothing, folds, flowers, etc. and the effect of one color on another will be noticeable without difficulty and the colors, as they appear under the circumstances can be located in the color pyramid.

§ 126

A special situation applies in case of polished objects, particularly for polished metallic vessels. Their color has a denseness and strength not available in painter's colors. As a result it would be futile to attempt to reproduce them with earth or juice colors. For this reason gold and silver are used when painting coats of arms. But coats of arms are only hieroglyphs, not to be considered in this connection. A painter has to represent metallic vessels in an entirely different manner. Of advantage to him is the fact that the more the vessels are shiny and polished the more they represent a kind of mirror in which other objects are reflected, if at times very distorted. If the vessel consists of polished silver the color of the reflected objects remains unchanged, except that they appear slightly weaker. Exact calculation of how the degree of strength depends on the angle of incidence is described in *Photometria* P. III, Chapter 1. Polished tin is more grayish in appearance and reflected objects appear darker and weaker than in polished silver. It is easily understood that much depends on the degree of flatness and polish. If the surface is weakly polished much white light is mixed into the color of the images, reflected from the uneven surface particles. As a result, the images appear lighter and in part more nebulous and this has to be taken into consideration by the painter. Polished iron and steel has a more bluish-black color and images of reflected objects are the more influenced by it, the poorer the polish is.

§ 127

All the changes in colors reflected from polished silver, tin, lead, iron, steel or other white mixtures of metals are limited if compared to brass, gold, copper, or other mixtures of colored metals. Regardless of their degree of polish their color is mixed with the color of objects mirrored in them. The degree of mixture depends to a considerable extent on the angle of incidence. If it is close to 90 degrees the color of the metal is more distinct than when the angle of incidence is less. At an angle of incidence of 1 to 2 degrees the color of the metal is absent or nearly so. The color of the metal declines approximately in line with the sine of the angle. The dependence of the color of the image, to the degree it is reflected, increases in different fashion that can be calculated according to the general formulas presented in Photometria (§ 433-434) once the portion reflected at a given angle has been experimentally determined. But if the metallic surface has only a weak polish the amount of color of the metal mixed in is higher and the image of objects appears less sharp, nebulous, in part also of more strongly changed color. The mixtures can be determined easily by drawing a line in the color pyramid between the color of the metal and the color of the object. Such a line will pass through the mixtures from which, according to the strength of the color of the metal and of the object, the appropriate one is to be selected.

§ 128

So far, I have treated the color pyramid as a just an ordered color chart, useful to easily find each color and to note its number and triangle. The problem of mixing the color itself points to another use of the pyramid. Trained color mixers or colorists know how to easily accomplish this using their normal colors. The advantage the pyramid offers them is that the colors they need to match are already present in it. As a result they can match them easier than if they are left to their own devises. For example, color # 16 needs to be matched. A first attempt results in color # 6 or # 15. It is easily apparent that to color # 6 1/8 part of yellow and 1/8 part of red, but to color # 15 only 1/8 part of yellow, need to be added to obtain color # 16. A more accurate calculation is not really necessary, but can be done easily. We have

in case of # 16: $b^1r^1g^6$
# 6: $b^3g^5$
# 15: $b^6r^3g^{15}$ .

These mixtures are to have equal parts of blue, with the result

# 16: $b^6r^6g^{36}$
# 6: $b^6g^{10}$
# 15: $b^6r^3g^{15}$.

It is now evident that if all three mixtures, as mixture # 16, need to be calculated in 1/36 parts then to mixture # 6 color $r^6g^{26}$ needs to be added and to mixture # 15 color $r^3g^{21}$ .

§ 129

In this way a colorist can make a test with his standard and as yet unmixed colors to determine how much of red, yellow, and blue each one contains. A simple comparison with the color pyramid is an easy path to achieve this. Using in a mixture more than three colors, or if white is included more than four colors, is an indication more of practical experience than of knowledge. The 8 or 9 colors that painters have commonly used up to now can in most cases be replaced by two colors. Extensive mixing only results in much trial and error, frequently consuming hours of time. The color pyramid is a much better guidepost compared to trial and error until the color is finally matched, whereupon it is no longer possible to say how the match is achieved.

§ 130

To produce a color by weighing the components is a bit involved if only a small amount is to be produced. For this reason it is useful to show how the mixture of the previously pasted pigments can be achieved with a paintbrush. I assume that only the three basic colors r, g, b will be used. Color # 27 is to be produced. It is located in the three rows # 1 to 42, # 25 to 30, # 6 to 36. It suffices to use one of these, say, the first one. Take a small amount of blue # 1 and add yellow # 9 to it until color # 42 is obtained. Add to this color # 42 the earlier found color # 3 until the desired color # 27 is matched. The basis of this approach is that in each color of the series # 3 to 42 the amount of yellow is the same and in actuality the mixture only progresses from red toward yellow. But the reason why it is better to first produce mixtures # 3 and # 42 before producing # 27 from these two is the fact that to mix a color from two components is much easier and faster than using immediately all three. However, also in that case the color pyramid can be used as an aid, as has been previously shown (§ 128). The quickest means is to prepare up front a stock of all the 45 colors of the lowest triangle. If they are to be applied in lighter versions white can be added to lighten the color.

§ 131

Incidentally, I have asked Mr. Calau to have twelve of the more beautiful painting pigments applied to the bottom frame of the pyramid so that they can be compared to the colors in the pyramid. From left to right they are the following:

1. Naples yellow
2. King's yellow or gold
3. Auripigment
4. Azurite
5. Smalt
6. Indigo
7. Lamp black
8. Juice green
9. Chrysocolla
10. Verdigris
11. Cinnabar
12. Florentine lake.

Most of them are opaque earth and metal colors. Except for that difference, similar colors appear among those of the pyramid. However, smalt and verdigris have a unique character resulting in specific uses for the two, but they are rather limited. Many of these colors do not mix well and this reduces their field of application considerably. To the degree that they can be mixed and used in oil painting what has earlier (§ 129) been proposed is the surest and simplest way to obtain knowledge of the colors resulting from their mixtures unless one wants, for the remainder of one's life, mix them by trial and error. Mr. **Calau,** who has paid much attention to all of these matters and has by himself reached many of the so far mentioned application conclusions of the color pyramid, has in this regard progressed already very far in the matter of oil and wax painting. Right at the start he produced two triangles using oil colors. For the first he used **Florentine lake**, **Berlin blue**, and **Naples yellow** and mixed some of his wax with them. The mixtures, made only at the 1/6 parts level, produced colors as he expected them. But it was apparent that colors # 9 – 45 were missing because **Florentine lake** is deviating quite significantly from **carmine** and **Naples yellow** is significantly below **gamboge**. The other triangle was based on yellow, copper red, and a bluish black. It is easily apparent that not all colors could be produced from these. It is only necessary to imagine a triangle where the corners fall on # 11, 17, 29. But the mixtures turned out well. Calau's wax always keeps them from deteriorating and protects the canvas against the corrosive effects of oils and partly also of the colors themselves.

§ 132

Illuminating or painting of copper engravings does not succeed very well if both juice colors as well as earth colors are used indiscriminately. In case of the latter it is better to have just contours of the figures in the copper engraving rather than detailed internal structures because they will just cover these up, as if they did not exist. Then shading has to be newly applied because the shading in the engraving is either partially or completely covered. Illumination is simpler with colors that can be applied as thinly as desired, because they remain transparent and lights and shadings already existing in the engraving are helpful even though it may be necessary to strengthen them, especially where colored light falls on shaded areas. The three basic colors carmine, Berlin blue, and gamboge serve best for these purposes because any other color can be matched by mixture and each mixture is transparent. I have already indicated (§ 111) to what degree, in most colors, the expensive carmine can be replaced with Florentine lake. Mr. **Calau** has also produced examples by painting, using the mentioned three colors, by already for producing several landscapes on gypsum disks and on *papier maché*. As well, he has illuminated several copper engravings where every object in light and in shade has its correct color. He showed that it is not necessary, as has been done in the past by painters of engravings, to paint roofs whose tiles are old and partly covered with moss with cinnabar. The pyramid has sufficient numbers of blackish, grayish, brownish, moss colors or tile colors to paint the tiles on old or new roofs, either in light or in shade, according to all changes in color. It is not problematic to paint the manifold of colors of plants, trees, birds, animals, clothing, rocks, types of earth, etc. because the pyramid, even if just taken as a sample collection, can offer all these colors from which to make appropriate choices.

§ 133

Because, as the pyramid demonstrates, all mixtures are produced with the three basic colors, it is sufficient to just acquire those. One of the advantages is that in several regards the color boxes, so far considered necessary, are no longer required. For purposes of travel pencils, brushes, paper, and the three basic colors suffice. In this color inventory the user has maximal flexibility for the case where each object is to be painted in its own color, or where only one mixture is to be used as an ink. Mr. **Calau** has already recognized this advantage. He grinds each of the three basic colors with the appropriate amount of wax and forms them into pastel crayons or tablets. These will be combined in the amounts to be used on a glass plate with water and an appropriate amount of gum and can then be used to make any desired mixture. Berlin blue, when ground only with water and gum has the tendency to become very hard when dried so that it needs to be ground again. But this is not necessary when it is combined with Calau's wax. I learned this from experience with the pastel crayons I purchased from Mr. **Calau**. If one end is made wet with water and that end rubbed against a glass plate the crayon can be its own rubbing stone and the color becomes as liquid as one wishes. The conclusion is that the particles of wax must place themselves between the particles of

Berlin blue so that the latter have little or no contact with themselves. As a result, they cannot be attached or glued to each other with such solidity that they can only be separated with force and much work.

§ 134

But this easy dissolvability of the colors combined with Calau's wax has another consequence that needs to be mentioned here. It is that such colors can also be easily dissolved with water after they have been applied on a painting. Of course, people do not paint with water colors so that they can afterwards brush over them with a wet sponge. But it must be mentioned here that also Calau's wax, when used in water colors, is subject to this problem. I say: when used with water colors. Because if the wax is used in oil colors, making the painting wet afterwards has no effect. But a painting with watercolors, when placed on a wet paper, can leave an imprint, if weakly colored. This situation has caused me to add a note to the book binder of this work so that he will take appropriate steps when including the color pyramid in the present book.

§ 135

The color pyramid makes it possible to understand without difficulty that Gauthier's art to print copperplates with three colors (§ 27) can be reasonably successful. The drawing is executed on three plates that are printed with red, yellow, and blue onto the same sheet. The lines of the three prints partly fall on top of each other and partly are placed side by side. The first case results in a directly mixed color. In the second one they appear to be mixed when viewed from an appropriate distance. If the plates are not engraved or etched but produced by the method of black art [book printing] the colors fall on top of each other and result in a mixed color. Such mixtures can be of any of the colors of the color pyramid depending on how each plate is more or less strongly shaded in the different areas of the picture. To obtain in this case specific results requires initial experiments, very similar to those described earlier in Chapter 7. Two copper plates are used. On each one squares of equal size are shaded according to different steps of strength, but in opposite order. The first plate is then, for example, printed with red and once the print has dried and is again moistened appropriately, the other plate is printed, for example with blue so that the stronger shaded squares fall on the weaker shaded red ones. From this it is possible to determine in which square the true middle color between red and blue appears. The same is done using red and yellow, and then also yellow and blue to again determine the middle color between them. It is important to determine at least eight visible steps of shading that differ to the same degree from one to the other. This will require additional testing. For example, on a plate with 9 squares the first is completely black and the ninth completely white. They are illuminated with amounts of light from 0 to 8, which is possible when using a lens with appropriately sized opening. In this way they should all appear equally black. Said differently, 8, 7, 6, 5, 4, 3, 2, 1, 0 parts of black are mixed with 0, 1, 2, 3, 4, 5, 6, 7, 8 parts of white and the result will be those steps that in the black art need to be produced in regard to strength and weakness of shading. These steps of shading are comparable to what I have presented in Chapter 7 concerning the weight of colors. If, for example, it is found that 5 grades of blue shade when printed together with 3 grades of red shade result in the middle color between red and blue it will be known what is required to obtain $r^4b^4$ in the lowest triangle. It is necessary to determine in the same manner the required degrees of shade for $r^4g^4$ and $g^4b^4$ if it is desired to obtain every mixture in the lowest triangle. For the higher triangles only 6, 4, 3, 2, 1, 0 portions of the three colors need to be used so that a correspondingly growing number of steps of white result. Also here it will be necessary to make a few preliminary trials in regard to the following. 1. Is it possible to find three printing inks for printing copper plates so that the mixtures are as complete as those between **carmine**, **Berlin blue**, and **gamboge**? 2. Are the result identical regardless of the sequence in which the three colors are printed? If the results differ one needs to select the sequence which produces the best results and then once and for all stick to it. Perhaps it will be found that when one color is printed on another the latter will be more or less glazed and possibly in part opaquely covered. For this reason, also in this situation much depends on the choice of the three basic colors so that, even when printed in a strong shade they maintain sufficient transparency. The fact that copper plates are cleaned after printing with lye and a varnish is added to the linseed oil and the possible resulting change in the colors must also be taken into consideration to be certain to later obtain all mixtures reproducibly. That this is important can be seen in the green colors of our color pyramid. In the outer rows they tend noticeably toward brown even though neither the blue nor the yellow used in these mixtures can be blamed for it. When weighing the colors for the mixtures each mixture was immediately applied to two hand-drawn pyramids on the same paper used for the printed version and the green colors looked perfect. That they look less pure after being applied to the pyramid printed from a copper plate must be due to that copper imprint.

§ 136

What I have mentioned here about the three basic colors in regard to copper engraving and the black art applies to cotton printing as well. Also here the important matter is the selection of the basic colors if, printed one on top of the other, they are to produce any mixture of colors. But just as in case of the copper engravings also in this case a number of preliminary experiments need to be made if the process is to be predictable in regard to number, measure, weight, and the portions necessary for any mixture. The issue of shading is particularly important if one wants to obtain all mixtures with three printing blocks.

§ 137

For special reasons it is useful to also discuss here the subject of writing inks. Mr. Lewis' history of colors [William Lewis, *Commercium philosophico-technium*, London: Baldwin, 1763, translated by J. H. Ziegler as *Historie der Farben*, Zürich: Heidegger, 1766] and especially of black, as well as my *Observations sur l'encre et le papier* (Observations regarding ink and paper), (Mémoires de l'Academie Royal de Berlin, 1770) provide me with the most immediate reason to do so. Common ink is made since centuries from oak gall and sulfuric acid. The ease of producing it and its blackness and liquidity have made it so popular that it is not only generally used but also has found application in textile dyeing. However, it was not realized that it fades with time and turns paper yellow so that old handwritings are after some time no longer readable. This has led Mr. Lewis on the one hand to improve the common ink and on the other to propose other more durable inks as well as more durable paper. He had knowledge of **Le Blon**'s and Father **Castell**'s mixture of black from other colors. But he mentioned that **Le Blon** did not indicate the required amounts of the components and that **Castell** demanded that black is not mixed from two or three colors, but from several of them. Perhaps **Lewis** did not take the time to consider the matter und to make corresponding experiments. He takes a black color made from black ink or from lampblack, etc. as being a very durable ink and only regrets that it can be rinsed of quite easily. It is true that the highly acidic common ink penetrates paper more deeply. But one cannot conclude from this that it is not possible to develop from other, much less salted, colored juices a permanent black ink that is strongly attached to paper. The possibility to produce it from two, three, or more colored juices in many different ways becomes clearly evident from the calculations made in the previous chapter. It is therefore just a matter of trials to determine the juices that result in a stable color, that mix well, and that themselves are stable without having to add salts to preserve the ink from degradation. It is these salts that over time move from the ink into the paper, thereby degrading it. They should not be present in the juices from the start. The manifold possibilities to make from yellow, red, and blue juices or those that approach them in color a black mixture or ink as well as inks of any other color means that one is not limited to just a few juices but that one can select from many those that produce the most beautiful and the most stable ink. All that is necessary is to compare the ink produced by any of the juices individually with the colors of the pyramid to find immediately which two, three, or four of them need to be mixed together so that the mixture is black or even if a black mixture can be produced from them.

§ 138

Finally, due to the fact that most of the colors in the bottom triangle are quite dark, I need to repeat the comment that, in order to distinguish between them, they need to be viewed in bright daylight. It is also helpful to use the higher triangles as an aid. The phrase I mentioned at the beginning (§ 11), "It will wish to be seen in the light," applies here to a high degree and it is evident from the comments made there that such darkness has its own special uses. The darkness is not just the result of the colors used, but mostly due to the admixture of **Calau**'s wax even though it is not only not black by itself but, when dissolved in water, is a milky white. The strength of the colors of the lowest triangle also is the reason that the cinnabar color is found in # 65 rather than in # 39 if the color is not applied particularly thinly. Finally, it is also easy to see that one and the same mixture will not have been applied in perfectly equal strength in all the copies of the pyramid. What I mentioned here and there about colors and their naming has to be understood with consideration of weaker or stronger intensity of application.

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## Note to the Bookbinder

The triangular chart of numbers printed on a separate page will be included at the end of the text. Behind it a white, quart-sized sheet of paper will be included to which the sheet containing the painted color pyramid will be glued and trimmed on all sides that, flat as it is, it can be folded into the book without a need to fold it further. The Pyramid is not to be glued in before the book is completely bound so that the colors do not by chance get wet and mark off when in the press. That the book is not to be beaten and that it should be trimmed as little as possible is self-evident.

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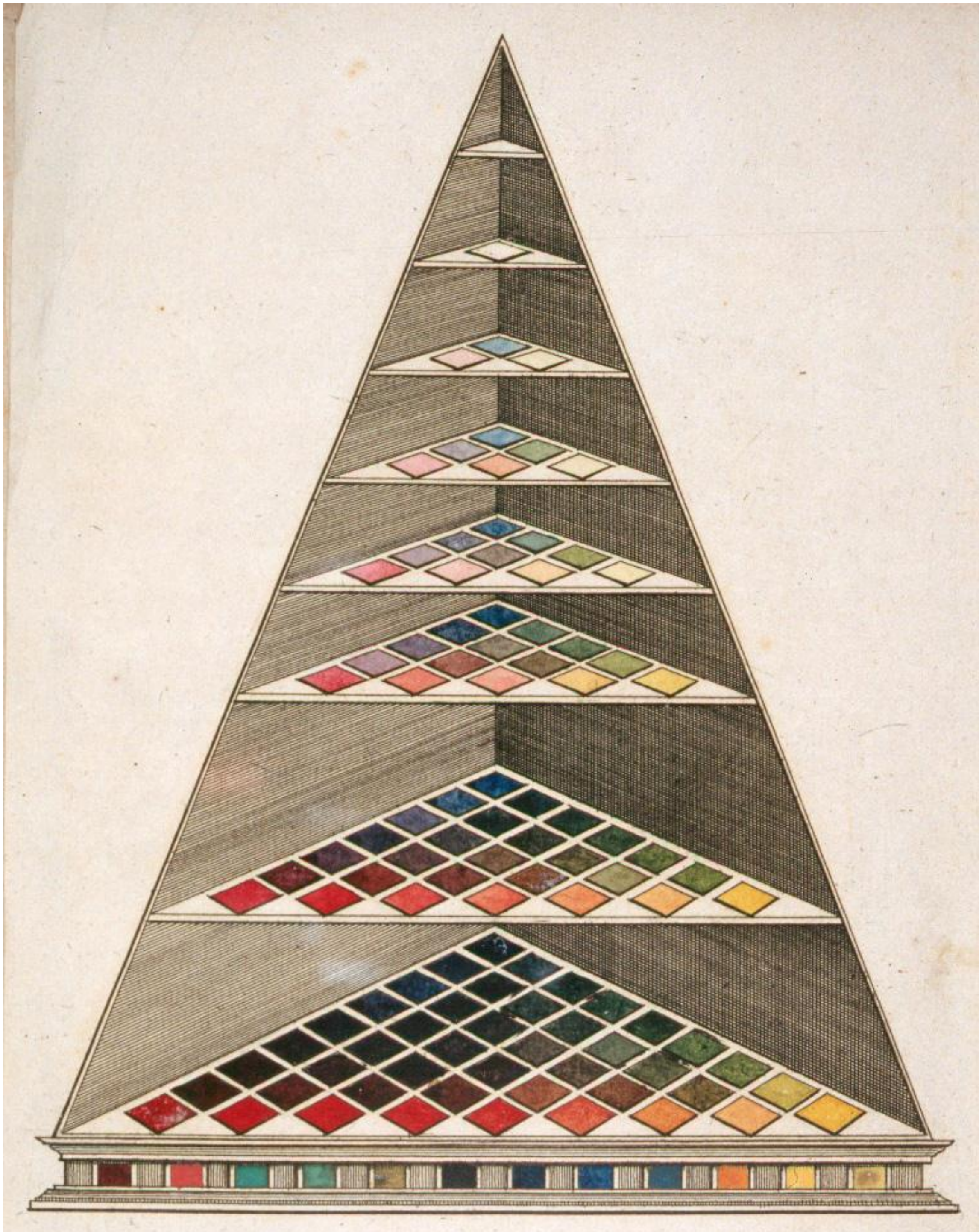

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48 58 63
9 60 65 45

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7 22 33 40
9 24 35 42 45

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46 50
47 51 56
5 52 57 31
48 53 58 61 63
49 54 59 62 64 66
9 55 60 35 65 67 45

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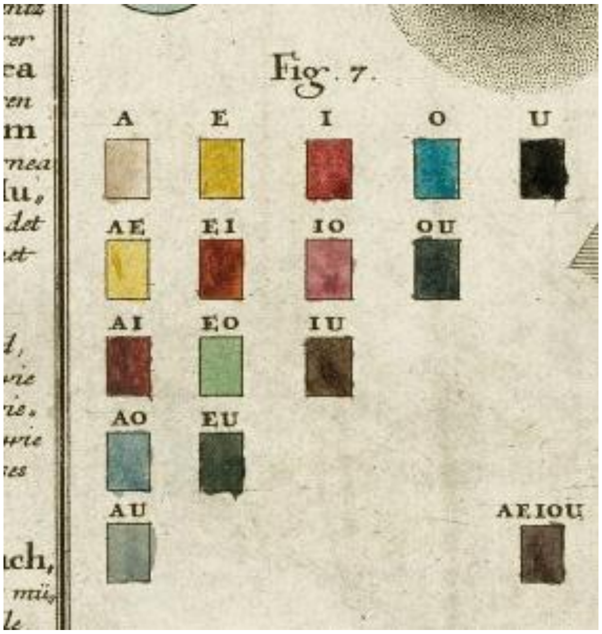
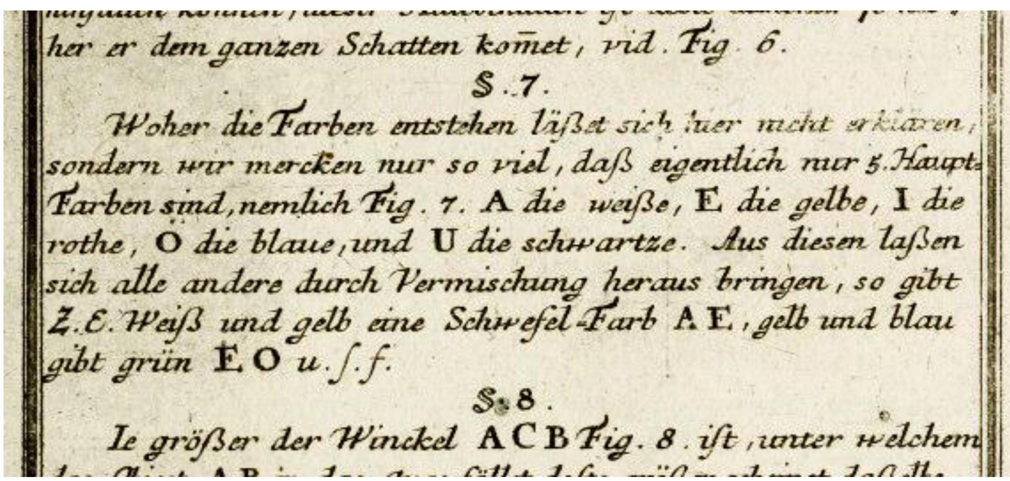
2 10
3 11 18
4 12 19 25
5 13 20 26 31
6 14 21 27 32 36
7 15 22 28 33 37 40
8 16 23 29 34 38 41 43
9 17 24 30 35 39 42 44 45

## Appendix

1. Tobias Mayer's first color mixture scheme from his *Mathematischer Atlas* (Augsburg, 1745), published when he was 22 years old. Page 51 contains Fig. 7 and its explanation.



her er dem ganzen Schatten komet, vid. Fig. 6.

§. 7.

Woher die Farben entstehen läßet sich hier nicht erklären, sondern wir mercken nur so viel, daß eigentlich nur 5. Haupt-Farben sind, nemlich Fig. 7. A die weiße, E die gelbe, I die rothe, O die blaue, und U die schwartze. Aus diesen laßen sich alle andere durch Vermischung heraus bringen, so gibt z.E. Weiß und gelb eine Schwefel-Farb A E, gelb und blau gibt grün E O u.s.f.

§. 8.

Ie größer der Winckel A C B Fig. 8. ist, unter welchem

Translation:
How colors are generated I cannot explain, but we notice that actually there are only 5 main colors, as shown in Fig. 7. A is the white color, E yellow, I red, O blue, and U the black color. From these all the others can be produced by mixture, for example white and yellow result in a sulfur color AE, yellow and blue produces green EO etc.

2. Tobias Mayer's central color triangle from *De affinitate colorum commentatio* (1758)